Answers - Integrals (Review Packet for Exam #2)

1. $$\int \frac{e^{4x}+2e^{2x}-e^{x}}{e^{2x}+1}\,dx = \int \frac{e^{x}(e^{3x}+2e^{x}-1)}{e^{2x}+1}\,dx = \int \frac{u^3+2u-1}{u^2+1}\,du = \int u + \frac{u-1}{u^2+1}\,du$$

let $\boxed{\begin{array}{l} u=e^x \\ du=e^x dx \end{array}}$

$$u^2+1\overline{\smash{)}u^3+2u-1}\quad\text{quotient } u;\quad \underline{u^3+u},\quad u-1$$

$$= \int u + \frac{u}{u^2+1} - \frac{1}{u^2+1}\,du$$

$$= \frac{u^2}{2} + \frac{1}{2}\ln|u^2+1| - \arctan u + C$$ ✓

$$= \boxed{\frac{e^{2x}}{2} + \frac{1}{2}\ln(e^{2x}+1) - \arctan e^x + C}$$

2. $$\int_0^3 \frac{1}{\sqrt{9-3x}}\,dx = \lim_{t\to 3}\int_0^t \frac{1}{\sqrt{9-3x}} = \lim_{t\to 3}\int_9^{9-3t} -\frac{1}{3}\,\frac{1}{\sqrt{u}}\,du$$

$u^{-1/2}$

$\boxed{\begin{array}{l} x=0 \Rightarrow u=9 \\ x=t \Rightarrow u=9-3t \end{array}}$ $\boxed{\begin{array}{l} u=9-3x \\ du=-3dx \\ -\frac{1}{3}du=dx \end{array}}$

$$= \lim_{t\to 3} -\frac{1}{3}\cdot 2u^{1/2}\Big|_9^{9-3t} = \lim_{t\to 3} -\frac{2}{3}\overset{\nearrow 0}{\sqrt{9-3t}} - \left(\frac{-2}{3}\overset{3}{\cancel{\sqrt{9}}}\right)$$

$= \boxed{2}$ Converges ✓

3. $$\int_1^\infty \frac{1}{3x+1}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{3x+1}\,dx = \lim_{t\to\infty}\frac{1}{3}\ln|3x+1|\Big|_1^t = \lim_{t\to\infty}\cancel{\frac{1}{3}\ln|3t+1|}^{\nearrow\infty} - \frac{1}{3}\ln|4| = \boxed{\infty}$$ Diverges ✓

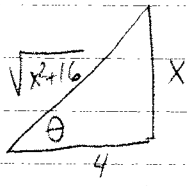
4. $$\int_3^\infty \frac{1}{(x^2+16)^{3/2}}\,dx = \lim_{t\to\infty}\int_3^t \frac{1}{(x^2+16)^{3/2}}\,dx = \lim_{t\to\infty}\int_{x=3}^{x=t}\frac{1}{\underbrace{(16\tan^2\theta+16)^{3/2}}_{(16\sec^2\theta)^{3/2}}}\cdot 4\sec^2\theta\,d\theta$$

save limits of integrat until the end

trig.sub. $\boxed{\begin{array}{l} x=4\tan\theta \\ dx=4\sec^2\theta d\theta \end{array}}$

$$= \lim_{t\to\infty}\int_{x=3}^{x=t}\frac{4\sec^2\theta\,d\theta}{(4\sec\theta)^3}$$

$$= \lim_{t\to\infty}\int_{x=3}^{x=t}\frac{1}{16\sec\theta}\,d\theta$$



$$= \lim_{t\to\infty}\int_{x=3}^{x=t}\frac{1}{16}\cos\theta\,d\theta$$

$\tan\theta = \frac{x}{4} \Rightarrow \sin\theta = \frac{x}{\sqrt{x^2+16}}$

$$= \lim_{t\to\infty}\frac{1}{16}\sin\theta\Big|_{x=3}^{x=t} = \lim_{t\to\infty}\frac{1}{16}\frac{x}{\sqrt{x^2+16}}\Big|_3^t$$

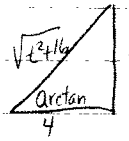
$$= \lim_{t\to\infty}\frac{1}{16}\frac{t}{\sqrt{t^2+16}} - \frac{3}{16\underset{5}{\sqrt{25}}}$$

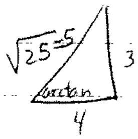
$\sqrt{\cancel{\frac{t^2}{t^2+16}}}$

$$= \frac{1}{16} - \frac{3}{5\cdot 16} = \frac{5-3}{80} = \frac{2}{80} = \boxed{\frac{1}{40}}$$ ✓ Converges

or 4
& switch limits of Integration

$$\lim_{t\to\infty}\int_{\arctan 3/4}^{\arctan t/4}\frac{1}{16}\cos\theta\,d\theta$$





$$\lim_{t\to\infty}\frac{1}{16}\sin\theta\Big|_{\arctan 3/4}^{\arctan t/4} = \lim_{t\to\infty}\frac{1}{16}\sin(\arctan t/4) - \frac{1}{16}\sin(\arctan 3/4)$$

$$= \lim_{t\to\infty}\frac{1}{16}\cancel{\frac{t}{\sqrt{t^2+16}}}^{\nearrow 1} - \frac{1}{16}\cdot\frac{3}{5} = \frac{1}{16}\left(1-\frac{3}{5}\right) = \frac{1}{16}\left(\frac{2}{5}\right) = \boxed{\frac{1}{40}}$$ ✓

5. $$\int_3^\infty \frac{1}{x^2-4x+7}\,dx = \lim_{t\to\infty}\int_3^t \frac{1}{x^2-4x+7}\,dx = \lim_{t\to\infty}\int_3^t \frac{1}{(x^2-4x+4)+3}\,dx = \lim_{t\to\infty}\int_3^t \frac{1}{(x-2)^2+3}\,dx$$

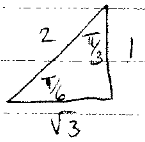
complete the square

$\boxed{\begin{array}{l} u=x-2 \\ du=dx \end{array}}$ $x=3 \Rightarrow u=1$

$x=t \Rightarrow u=t-2$

$$= \lim_{t\to\infty}\int_1^{t-2} \frac{1}{u^2+3}\,du = \lim_{t\to\infty}\int_1^{t-2} \frac{1}{3}\,\frac{1}{\left(\frac{u}{\sqrt{3}}\right)^2+1}\,du = \lim_{t\to\infty} \frac{1}{3}\sqrt{3}\int_{1/\sqrt{3}}^{\frac{t-2}{\sqrt{3}}} \frac{1}{w^2+1}\,dw$$

$\boxed{\begin{array}{l} w=\frac{u}{\sqrt{3}} \\ dw=1/\sqrt{3}\,du \\ \sqrt{3}\,dw=du \end{array}}$ $\boxed{\begin{array}{l} u=1 \Rightarrow w=1/\sqrt{3} \\ u=t-2 \Rightarrow w=\frac{t-2}{\sqrt{3}} \end{array}}$



$$= \lim_{t\to\infty} \frac{1}{\sqrt{3}} \arctan w \Big|_{1/\sqrt{3}}^{\frac{t-2}{\sqrt{3}}} = \lim_{t\to\infty} \frac{1}{\sqrt{3}} \overset{\pi/2}{\cancel{\arctan\left(\frac{t-2}{\sqrt{3}}\right)}} - \frac{1}{\sqrt{3}} \overset{\pi/6}{\cancel{\arctan\frac{1}{\sqrt{3}}}}$$

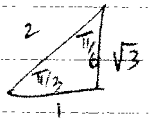
$$= \frac{1}{\sqrt{3}}\cdot\pi/2 - \frac{1}{\sqrt{3}}\cdot\pi/6 = \frac{1}{\sqrt{3}}\left(\frac{3\pi-\pi}{6}\right) = \frac{1}{\sqrt{3}}\,2\pi/6 = \boxed{\frac{\pi}{3\sqrt{3}}}$$ ✓ Converges.

6. $$\int_e^\infty \frac{1}{x(\ln x)^3}\,dx = \lim_{t\to\infty}\int_e^t \frac{1}{x(\ln x)^3}\,dx = \lim_{t\to\infty}\int_1^{\ln t} \underset{u^{-3}}{\frac{1}{u^3}}\,du = \lim_{t\to\infty} \frac{u^{-2}}{-2}\Big|_1^{\ln t} = \lim_{t\to\infty} \overset{0}{\cancel{\frac{-1}{2(\ln t)^2}}} - \left(\frac{-1}{2}\right)$$

$\boxed{\begin{array}{ll} u=\ln x & x=e \Rightarrow u=\ln e=1 \\ du=1/x\,dx & x=t \Rightarrow u=\ln t \end{array}}$

$= \boxed{1/2}$ ✓ Converges

7. $$\int_0^3 \frac{\arctan\sqrt{x}}{\sqrt{x}(1+x)}\,dx = \lim_{t\to0^+}\int_t^3 \frac{\arctan\sqrt{x}}{\sqrt{x}\,(1+(\sqrt{x})^2)}\,dx = \lim_{t\to0^+} 2\int_{\arctan 3}^{\arctan\sqrt{t}} u\,du = \lim_{t\to0^+} u^2\Big|_{\arctan\sqrt{t}}^{\pi/3}$$



$\boxed{\begin{array}{l} u=\arctan\sqrt{x} \\ du=\frac{1}{(\sqrt{x})^2+1}\,\frac{1}{2\sqrt{x}}\,dx \\ 2du=\frac{1}{[(\sqrt{x})^2+1]\sqrt{x}}\,dx \end{array}}$ $\boxed{\begin{array}{l} x=t \Rightarrow u=\arctan\sqrt{t} \\ x=3 \Rightarrow u=\arctan\sqrt{3}=\pi/3 \end{array}}$

$$= \lim_{t\to0^+} \left(\pi/3\right)^2 - \left(\overset{0}{\cancel{\arctan\sqrt{t}}}\right)^2$$

$$= \boxed{\pi^2/9}$$ ✓ converges

8. $$\int_0^\infty \frac{1}{(x+2)(2x+5)}\,dx = \lim_{t\to\infty}\int_0^t \frac{1}{(x+2)(2x+5)}\,dx = \lim_{t\to\infty}\int_0^t \frac{1}{x+2} - \frac{2}{2x+5}\,dx = \lim_{t\to\infty} \ln|x+2| - \ln|2x+5|\,\Big|_0^t$$

Decompose - Partial Fractions

$$\frac{1}{(x+2)(2x+5)} = \frac{A}{x+2} + \frac{B}{2x+5}\Bigg]\,(x+2)(2x+5)$$

$1 = A(2x+5) + B(x+2)$

$= 2Ax + 5A + Bx + 2B$

$= (2A+B)x + (5A+2B)$

$\Rightarrow 2A+B=0 \Rightarrow B=-2A$

$5A+2B=1$

$5A+2(-2A)=1$

$5A-4A=1 \Rightarrow A=1 \Rightarrow B=-2$

$$= \lim_{t\to\infty} \ln|t+2| - \ln|2t+5| - (\ln 2 - \ln 5)$$

$$= \lim_{t\to\infty} \ln \overset{1/2}{\cancel{\frac{t+2}{2t+5}}} - \ln 2 + \ln 5$$

$$= \overset{0}{\cancel{\ln 1}} - \ln 2 - \ln 2 + \ln 5$$

$$= -2\ln 2 + \ln 5 = -\ln 2^2 + \ln 5 = -\ln 4 + \ln 5$$

$$= \boxed{\ln(5/4)}$$ Converges

u-sub

complete square to arc tan

$$9.\ \int \frac{2x^2-2x+6}{(x-1)\underbrace{(x^2-2x+7)}_{\text{irreducible}}}\,dx = \int \frac{1}{x-1} + \overset{\frac{x-1+2}{x^2-2x+7}}{\frac{x+1}{\underset{2x-2}{x^2-2x+7}}}\,dx = \ln|x-1| + \int \frac{x-1}{x^2-2x+7}\,dx + \int \frac{2}{x^2-2x+7}\,dx$$

Decompose - Partial Fractions

$$\left[\frac{2x^2-2x+6}{(x-1)(x^2-2x+7)} = \frac{A}{x-1} + \frac{Bx+C}{x^2-2x+7}\right](x-1)(x^2-2x+7)$$

$$2x^2-2x+6 = A(x^2-2x+7) + (Bx+C)(x-1)$$

$$= Ax^2-2Ax+7A+Bx^2+Cx-Bx-C$$

$$= (A+B)x^2 + (-2A-B+C)x + 7A-C$$

$\Rightarrow$ • $A+B=2$ $\quad B=2-A$

• $-2A-B+C=-2$ $\quad -2A-(2-A)+(7A-6)=-2$

• $7A-C=6$ $\quad -2A-\cancel{2}+A+7A-6=\cancel{-2}$

$\hookrightarrow C=7A-6$

$+6A=6$

$A=1$

$\Rightarrow B=1$

$\Rightarrow C=7-6=1$

$u=x^2-2x+7$

$du=2x-2\,dx$

$\frac12 du = x-1\,dx$

$$+\frac12\int\frac1u\,du$$

$$+\frac12\ln|u|$$

$$+\frac12\ln|x^2-2x+7|$$

$$+\int\frac{2}{(x-1)^2+6}\,dx$$

$w=x-1$

$dw=dx$

$$+2\int\frac{1}{w^2+6}\,dw$$

$$+\frac26\int\frac{1}{\left(\frac{w}{\sqrt6}\right)^2+1}\,dw$$

$v=w/\sqrt6$

$dv=\frac{1}{\sqrt6}dw$

$\sqrt6\,dv=dw$

$$+\frac26\cdot\sqrt6\int\frac{1}{v^2+1}\,dv$$

$$+\frac{2}{\sqrt6}\arctan v$$

$$+\frac{2}{\sqrt6}\arctan\left(\frac{x-1}{\sqrt6}\right)+($$

piece together $\boxed{\ln|x-1| + \frac12\ln|x^2-2x+7| + \frac{2}{\sqrt6}\arctan\left(\frac{x-1}{\sqrt6}\right)+C}$

$$10.\ \int_7^\infty \frac{1}{x^2-8x+19}\,dx = \lim_{t\to\infty}\int_7^t \frac{1}{(x^2-8x+16)+3} = \lim_{t\to\infty}\int_7^t\frac{1}{(x-4)^2+3}\,dx = \lim_{t\to\infty}\int_3^{t-4}\frac{1}{u^2+3}\,du$$

complete the square

$u=x-4$	$x=7 \Rightarrow u=3$
$du=dx$	$x=t \Rightarrow u=t-4$

$$=\lim_{t\to\infty}\frac13\int_3^{t-4}\frac{1}{\left(\frac{u}{\sqrt3}\right)^2+1}\,du$$

$w=\frac{u}{\sqrt3}$

$dw=\frac{1}{\sqrt3}du$

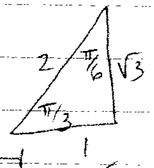
$\sqrt3\,dw=du$

$u=3 \Rightarrow w=\sqrt3$

$u=t-4 \Rightarrow w=\frac{t-4}{\sqrt3}$

$$=\lim_{t\to\infty}\frac{\sqrt3}{3}\int_{\sqrt3}^{\frac{t-4}{\sqrt3}}\frac{1}{w^2+1}\,dw = \lim_{t\to\infty}\frac{1}{\sqrt3}\arctan w\Big|_{\sqrt3}^{t-4/\sqrt3}$$

$$=\lim_{t\to\infty}\frac{1}{\sqrt3}\overset{\pi/2}{\arctan\left(\frac{t-4}{\sqrt3}\right)} - \frac{1}{\sqrt3}\overset{\pi/3}{\arctan\sqrt3}$$



$$= \frac{1}{\sqrt3}\left(\pi/2 - \pi/3\right) = \frac{1}{\sqrt3}\left(\frac{3\pi}{6}-\frac{2\pi}{6}\right) = \boxed{\frac{\pi}{6\sqrt3}}$$ ✓ Converges

$$11.\ \int_0^1 \frac{\ln x}{\sqrt x}\,dx = \lim_{t\to0^+}\int_t^1\frac{\ln x}{\sqrt x}\,dx = \lim_{t\to0^+} 2\sqrt x\ln x\Big|_t^1 - \int_t^1 2x^{1/2}\frac1x\,dx = \lim_{t\to0^+}2\sqrt x\ln x\Big|_t^1 - 2\int_t^1 x^{-1/2}\,dx$$

$u=\ln x$	$dv=x^{-1/2}dx$
$du=\frac1x dx$	$v=2x^{1/2}$

$$=\lim_{t\to0^+} 2\sqrt x\ln x - 4\sqrt x\Big|_t^1$$

$$=\lim_{t\to0^+}(2\cancel{\ln 1}^{\,0}-4) - (\underbrace{\cancel{2\sqrt t\ln t}^{\,\to0}}_{(*)} - \cancel{4\sqrt t}^{\,\to0}) = \boxed{-4}$$ ✓ converges.

(*)

$$\underset{0\cdot(\infty)}{\lim_{t\to0^+}\sqrt t\ln t} = \lim_{t\to0^+}\frac{\ln t}{t^{-1/2}} \overset{\frac{-\infty}{\infty}\,\text{L'H}}{=} \lim_{t\to0^+}\frac{\frac1t}{-\frac12 t^{-3/2}} = \lim_{t\to0^+}\cancel{-2t^{1/2}}^{\,\to0} = 0$$

$$12.\ \int \frac{1}{(x+3)(3x+1)}dx = \int \frac{-1/8}{x+3} + \frac{\frac{3}{8}}{3x+1}\,dx = -\frac{1}{8}\ln|x+3| + \frac{\cancel{3}}{8}\frac{\ln|3x+1|}{\cancel{3}} + C$$

$$= \boxed{-\frac{1}{8}\ln|x+3| + \frac{1}{8}\ln|3x+1| + C}$$

Decompose - Partial Fractions

$$\left(\frac{1}{(x+3)(3x+1)} = \frac{A}{x+3} + \frac{B}{3x+1}\right)(x+3)(3x+1)$$

$1 = A(3x+1) + B(x+3)$

$= (3A+B)x + (A+3B)$

$\Rightarrow$ • $3A+B=0$ $\quad B=-3A$

• $A+3B=1$ $\quad A-9A=1$

$-8A=1$

$\Rightarrow A=-1/8$

$\Rightarrow B=3/8$

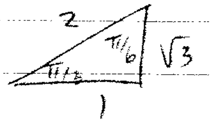
$$13.\ \int_{12}^{\infty} \frac{1}{x\sqrt{x-3}}dx = \lim_{t\to\infty}\int_{12}^{t}\frac{1}{x\sqrt{x-3}}dx = \lim_{t\to\infty}\int_{9}^{t-3}\frac{1}{(u+3)\sqrt{u}}du = \lim_{t\to\infty}2\int_{9}^{t-3}\frac{1}{((\sqrt{u})^2+3)\sqrt{u}}du$$

$u=x-3 \Rightarrow u+3=x$ $du=dx$	$x=12 \Rightarrow u=9$ $x=t \Rightarrow u=t-3$

$w=\sqrt{u}$ $dw=\frac{1}{2\sqrt{u}}du$ $2dw=\frac{1}{\sqrt{u}}du$	$u=9 \Rightarrow w=3$ $u=t-3 \Rightarrow w=\sqrt{t-3}$

$$= \lim_{t\to\infty}2\int_{3}^{\sqrt{t-3}}\frac{1}{w^2+3}dw = \lim_{t\to\infty}\frac{2}{3}\int_{3}^{\sqrt{t-3}}\frac{1}{\left(\frac{w}{\sqrt{3}}\right)^2+1}dw = \lim_{t\to\infty}\frac{2\sqrt{3}}{3}\int_{\sqrt{3}}^{\sqrt{\frac{t-3}{3}}}\frac{1}{v^2+1}dv$$

$v=\frac{w}{\sqrt{3}}$ $dv=\frac{1}{\sqrt{3}}dw$ $\sqrt{3}dv=dw$	$w=3 \Rightarrow v=\sqrt{3}$ $w=\sqrt{t-3} \Rightarrow v=\frac{\sqrt{t-3}}{\sqrt{3}}$



$$= \lim_{t\to\infty}\frac{2}{\sqrt{3}}\arctan v\Big|_{\sqrt{3}}^{\sqrt{\frac{t-3}{3}}} = \lim_{t\to\infty}\frac{2}{\sqrt{3}}\overset{\pi/2}{\arctan\sqrt{\frac{t-3}{3}}} - \frac{2}{\sqrt{3}}\overset{\pi/3}{\arctan\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}\left(\frac{\pi}{2}-\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}\left(\frac{3\pi}{6}-\frac{2\pi}{6}\right) = \frac{2}{\sqrt{3}}\left(\frac{\pi}{6}\right) = \boxed{\frac{\pi}{3\sqrt{3}}}\ ✓$$

converges

$x^2-2x+4 = x^2-2x+1+3 = (x-1)^2+3$

14. $\int_2^\infty \frac{1}{x^2-2x+4}dx = \lim_{t\to\infty}\int_2^t \frac{1}{(x-1)^2+3}dx = \lim_{t\to\infty}\int_1^{t-1}\frac{1}{u^2+3}du = \lim_{t\to\infty}\frac{1}{3}\int_1^{t-1}\frac{1}{\left(\frac{u}{\sqrt3}\right)^2+1}du$

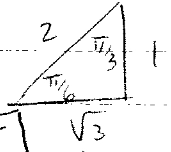
irreducible
complete the square

$$\begin{array}{|l|l|}\hline u=x-1 & x=2 \Rightarrow u=1 \\ du=dx & x=t \Rightarrow u=t-1 \\ \hline \end{array}$$

$$\begin{array}{|l|l|}\hline w=\frac{u}{\sqrt3} & u=1 \Rightarrow w=\frac{1}{\sqrt3} \\ dw=\frac{1}{\sqrt3}du & u=t-1 \Rightarrow w=\frac{t-1}{\sqrt{}} \\ \sqrt3\,dw=du & \\ \hline \end{array}$$

$= \lim_{t\to\infty}\frac{\sqrt3}{3}\int_{1/\sqrt3}^{\frac{t-1}{\sqrt3}}\frac{1}{w^2+1}dw = \lim_{t\to\infty}\frac{1}{\sqrt3}\arctan w\Big|_{1/\sqrt3}^{\frac{t-1}{\sqrt3}}$

$= \lim_{t\to\infty}\frac{1}{\sqrt3}\overset{\nearrow\pi/2}{\arctan\left(\frac{t-1}{\sqrt3}\right)} - \frac{1}{\sqrt3}\overset{\nearrow\pi/6}{\arctan\frac{1}{\sqrt3}}$



$= \frac{1}{\sqrt3}\left[\frac{\pi}{2}-\frac{\pi}{6}\right] = \frac{1}{\sqrt3}\left[\frac{3\pi}{6}-\frac{\pi}{6}\right] = \frac{1}{\sqrt3}\cdot\frac{\pi}{3} = \boxed{\frac{\pi}{3\sqrt3}}$ ✓ Converges

15. $\int\frac{1}{x^2+2x+2}dx = \int\frac{1}{(x^2+2x+1)+1}dx = \int\frac{1}{(x+1)^2+1}dx = \int\frac{1}{u^2+1}du = \arctan u + C = \boxed{\arctan(x+1)+C}$ ✓

complete the square

$u=x+1$
$du=dx$

16. $\int_1^\infty\frac{\sqrt x}{1+x^3}dx = \lim_{t\to\infty}\int_1^t\frac{\sqrt x}{1+x^3}dx = \lim_{t\to\infty}\int_1^t\frac{\sqrt x}{1+(x^{3/2})^2}dx = \lim_{t\to\infty}\frac{2}{3}\int_1^{t^{3/2}}\frac{1}{1+u^2}du$

$$\begin{array}{|l|}\hline u=x^{3/2} \\ du=\frac{3}{2}x^{1/2}dx \\ \frac{2}{3}du=x^{1/2}dx \\ \hline \end{array} \quad \begin{array}{|l|}\hline x=1 \Rightarrow u=1 \\ x=t \Rightarrow u=t^{3/2} \\ \hline \end{array}$$

$= \lim_{t\to\infty}\frac{2}{3}\arctan u\Big|_1^{t^{3/2}} = \lim_{t\to\infty}\frac{2}{3}\overset{\nearrow\pi/2}{\arctan t^{3/2}} - \frac{2}{3}\overset{\nearrow\pi/4}{\arctan 1}$

$= \frac{2}{3}\left[\frac{\pi}{2}-\frac{\pi}{4}\right] = \frac{2}{3}\left[\frac{\pi}{4}\right] = \boxed{\frac{\pi}{6}}$ ✓ converges

17. $\int_0^4\frac{1}{(8-2x)^{1/3}}dx = \lim_{t\to4^-}\int_0^t\frac{1}{(8-2x)^{1/3}} = \lim_{t\to4^-}-\frac{1}{2}\int_8^{8-2t}\frac{1}{u^{1/3}}du = \lim_{t\to4^-}-\frac{1}{2}\cdot\frac{3}{2}u^{2/3}\Big|_8^{8-2t}$

$u^{-1/3}$

$u=8-2x$
$du=-2dx$
$-\frac{1}{2}du=dx$

$x=0 \Rightarrow u=8$
$x=t \Rightarrow u=8-2t$

$= \lim_{t\to4^-}\overset{\nearrow0}{-\frac{3}{4}(8-2t)^{2/3}} - \overset{\nearrow4}{\left(-\frac{3}{4}\cdot8^{2/3}\right)}$

$= \boxed{+3}$ ✓ Converges

18. $$\int \frac{1}{-x^2+2x+3}\,dx = -\int \frac{1}{x^2-2x-3}\,dx - \int \frac{1}{(x-3)(x+1)}\,dx = -\int \frac{\frac{1}{4}}{x-3} + \frac{-1/4}{x+1}\,dx$$

Decompose - Partial Fractions

$$\left[\frac{1}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}\right](x-3)(x+1)$$

$$\Rightarrow 1 = A(x+1) + B(x-3)$$

$$= (A+B)x + A - 3B$$

$$\Rightarrow \bullet\ A+B=0 \Rightarrow B=-A$$

$$\bullet\ A-3B=1 \quad A-3(-A)=1$$

$$4A=1$$

$$A=1/4$$

$$\Rightarrow B=-1/4$$

$$= -\frac{1}{4}\int \frac{1}{x-3} - \frac{1}{x+1}\,dx$$

$$\boxed{= -\frac{1}{4}\left[\ln|x-3| - \ln|x+1|\right] + C}$$

$$= \boxed{-\frac{1}{4}\ln\left|\frac{x-3}{x+1}\right| + C}$$ ✓

or $$\boxed{\frac{1}{4}\ln\left|\frac{x+1}{x-3}\right| + C}$$

19. $$\int_2^\infty \frac{1}{(x^2+4)^2}\,dx = \lim_{t\to\infty}\int_2^t \frac{1}{(x^2+4)^2}\,dx = \lim_{t\to\infty}\int_{\pi/4}^{\arctan t/2} \frac{1}{[\underbrace{4\tan^2\theta+4}_{4\sec^2\theta}]^2}\,2\sec^2\theta\,d\theta$$

$$\boxed{\begin{array}{l|l} x = 2\tan\theta & x=2 \Rightarrow \theta = \arctan 1 = \pi/4 \\ dx = 2\sec^2\theta\,d\theta & x=t \Rightarrow \theta = \arctan t/2 \end{array}}$$

$$= \lim_{t\to\infty}\int_{\pi/4}^{\arctan t/2} \frac{2\sec^2\theta\,d\theta}{\underbrace{16\sec^4\theta}_{\frac{1}{\sec^2\theta}}} = \lim_{t\to\infty}\frac{1}{8}\int_{\pi/4}^{\arctan t/2}\cos^2\theta\,d\theta$$

$$= \lim_{t\to\infty}\frac{1}{8}\int_{\pi/4}^{\arctan t/2}\frac{1+\cos(2\theta)}{2}\,d\theta = \lim_{t\to\infty}\frac{1}{16}\left[\theta + \frac{\overset{2\sin\theta\cos\theta}{\sin(2\theta)}}{\cancel{2}}\right]\Bigg|_{\pi/4}^{\arctan t/2}$$

$$= \lim_{t\to\infty}\frac{1}{16}\left[\left(\overset{\pi/2}{\arctan t/2} + \sin\Big(\underbrace{2\,\overset{\pi/2}{\arctan(t/2)}}_{\sin\pi \to 0}\Big)\right) - \left(\pi/4 + \frac{\overset{\sin\pi/2=1}{\sin(2\cdot\pi/4)}}{2}\right)\right]$$ ✓

$$= \frac{1}{16}\left[\pi/2 + 0 - \pi/4 - 1/2\right] = \frac{1}{16}\left[\pi/4 - 1/2\right] = \frac{1}{32}\left[\pi/2 - 1\right] = \boxed{\frac{1}{64}\left[\pi - 2\right]}$$

converges

20. $$\int_{-1}^{1}\frac{1}{\sqrt{1-x^2}}\,dx=\int_{-1}^{0}\frac{1}{\sqrt{1-x^2}}\,dx+\int_{0}^{1}\frac{1}{\sqrt{1-x^2}}\,dx$$

$$=\lim_{s\to -1^+}\int_{s}^{0}\frac{1}{\sqrt{1-x^2}}\,dx+\lim_{t\to 1^-}\int_{0}^{t}\frac{1}{\sqrt{1-x^2}}\,dx$$

$$=\lim_{s\to -1^+}\arcsin x\Big|_{s}^{0}+\lim_{t\to 1^-}\arcsin x\Big|_{0}^{t}$$

$$=\lim_{s\to -1^+}\cancelto{0}{\arcsin 0}-\cancelto{-\pi/2}{\arcsin s}+\lim_{t\to 1^-}\arcsin t-\cancelto{0}{\arcsin 0}$$

$$=-\left(-\tfrac{\pi}{2}\right)+\tfrac{\pi}{2}=\boxed{\pi}$$ ✓ converges

OR notice even function, so compute half $\int_0^1\frac{1}{\sqrt{1-x^2}}\,dx=\pi/2$ and double.

21. $$\int_{0}^{1}\frac{1}{\sqrt{x}}\,dx=\lim_{t\to 0^+}\int_{t}^{1}x^{-1/2}\,dx=\lim_{t\to 0^+}2\sqrt{x}\Big|_{t}^{1}=\lim_{t\to 0^+}2-\cancelto{0}{2\sqrt{t}}=\boxed{2}$$ ✓ Converges

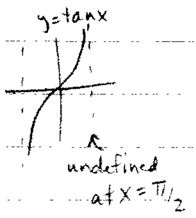
22. $$\int_{0}^{1}\frac{1}{x}\,dx=\lim_{t\to 0^+}\int_{t}^{1}\frac{1}{x}\,dx=\lim_{t\to 0^+}\ln|x|\Big|_{t}^{1}=\lim_{t\to 0^+}\cancelto{0}{\ln 1}-\cancelto{-\infty}{\ln t}=\boxed{+\infty}$$ ✓ Diverges

23. $$\int_{1}^{\infty}\frac{1}{x}\,dx=\lim_{t\to\infty}\int_{1}^{t}\frac{1}{x}\,dx=\lim_{t\to\infty}\ln|x|\Big|_{1}^{t}=\lim_{t\to\infty}\ln|t|-\cancelto{0}{\ln 1}=\boxed{\infty}$$ ✓ Diverges

24. $$\int_{0}^{1}\frac{1}{x^2}\,dx=\lim_{t\to 0^+}\int_{t}^{1}\underset{x^{-2}}{\frac{1}{x^2}}\,dx=\lim_{t\to 0^+}-x^{-1}\Big|_{t}^{1}=\lim_{t\to 0^+}\frac{-1}{1}-\left(-\frac{1}{t}\right)$$

$$=\lim_{t\to 0^+}-1+\cancelto{+\infty}{\frac{1}{t}}=\boxed{\infty}$$ ✓ Diverges.

25. $$\int_{1}^{\infty}\frac{1}{x^2}\,dx=\lim_{t\to\infty}\int_{1}^{t}x^{-2}\,dx=\lim_{t\to\infty}\frac{x^{-1}}{-1}\Big|_{1}^{t}=\lim_{t\to\infty}\cancelto{0}{\frac{-1}{t}}-\left(-\frac{1}{1}\right)=\boxed{1}$$ ✓ Converges



26. $$\int_{0}^{\pi/2}\tan x\,dx=\lim_{t\to\pi/2^-}\int_{0}^{t}\tan x\,dx=\lim_{t\to\pi/2^-}-\ln|\cos x|\Big|_{0}^{t}=\lim_{t\to\pi/2^-}-\cancelto{-\infty}{\ln|\cancelto{0^+}{\cos t}|}-\left(-\cancelto{0}{\ln\cos 0}\right)$$

$$=\boxed{+\infty}$$ Diverges.

$$27\ \int_0^1 \frac{1-2x}{\sqrt{x-x^2}}dx = \int_0^1 \frac{1-2x}{\sqrt{x(1-x)}}dx = \int_0^{1/2} \frac{1-2x}{\sqrt{x-x^2}}dx + \int_{1/2}^{1} \frac{1-2x}{\sqrt{x-x^2}}dx$$

$u = x-x^2$

$du = 1-2x\,dx$

undef. at $x=0$ and $x=1$

$$= \lim_{s\to 0^+} \int_s^{1/2} \frac{1-2x}{\sqrt{x-x^2}}dx + \lim_{t\to 1^-} \int_{1/2}^{t} \frac{1-2x}{\sqrt{x-x^2}}dx$$

$$= \lim_{s\to 0^+} \int_{s-s^2}^{1/4} \frac{1}{\sqrt{u}}du + \lim_{t\to 1^-} \int_{1/4}^{t-t^2} \frac{1}{\sqrt{u}}du$$

$x = 1/2$

$\Rightarrow u = 1/2 \cdot 1/4 = 1/4$

$$= \lim_{s\to 0^+} 2\sqrt{u}\Big|_{s-s^2}^{1/4} + \lim_{t\to 1^-} 2\sqrt{u}\Big|_{1/4}^{t-t^2}$$

$$= \lim_{s\to 0^+} 2\sqrt{1/4} - \overset{0}{\cancel{2\sqrt{s-s^2}}} + \lim_{t\to 1^-} \overset{0}{\cancel{2\sqrt{t-t^2}}} - 2\sqrt{1/4}$$

$$= 2\cdot 1/2 - 2\cdot 1/2 = \boxed{0}$$ ✓ converges

$$28.\ \int_0^\infty e^{-x}dx = \lim_{t\to\infty} \int_0^t e^{-x}dx = \lim_{t\to\infty} -e^{-x}\Big|_0^t = \lim_{t\to\infty} -\overset{0}{\cancel{e^{-t}}} - (-\overset{1}{\cancel{e^{0}}}) = \boxed{1}$$ ✓ converges.

$$29\ \int_0^{\pi/2} \sec^2 x\,dx = \lim_{t\to \pi/2^-} \int_0^t \sec^2 x\,dx = \lim_{t\to\pi/2^-} \tan x\Big|_0^t = \lim_{t\to\pi/2^-} \tan t - \overset{0}{\cancel{\tan 0}} = \boxed{\infty}$$ ✓ Diverges

$$30.\ \int_3^4 \frac{1}{(x-4)^2}dx = \lim_{t\to 4^-} \int_3^t \frac{1}{(x-4)^2}dx = \lim_{t\to 4^-} \int_{-1}^{t-4} u^{-2}du = \lim_{t\to 4^-} -u^{-1}\Big|_{-1}^{t-4} = \lim_{t\to 4^-} \frac{-1}{u}\Big|_{-1}^{t-4}$$

$u = x-4$	$x=3 \Rightarrow u=-1$
$du = dx$	$x=t \Rightarrow u=t-4$

$$= \lim_{t\to 4^-} \frac{-1}{t-4} - \left(\frac{-1}{-1}\right)$$

$$= -\frac{1}{0^-} - 1 = \boxed{+\infty}$$ ✓ Diverges

$$31.\ \int_1^2 \frac{1}{x\ln x}dx = \lim_{t\to 1^+} \int_t^2 \frac{1}{x\ln x}dx = \lim_{t\to 1^+} \int_{\ln t}^{\ln 2} \frac{1}{u}du = \lim_{t\to 1^+} \ln|u|\Big|_{\ln t}^{\ln 2}$$

$u = \ln x$	$x=t \Rightarrow u=\ln t$
$du = \frac{1}{x}dx$	$x=2 \Rightarrow u=\ln 2$

$$= \lim_{t\to 1^+} \ln|\ln 2| - \ln|\overset{0^+}{\cancel{\ln t}}|$$

$$= \ln\ln 2 - (-\infty) = \boxed{+\infty}$$ ✓ Diverges.

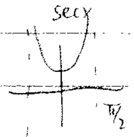
32. $\int_{-\pi/2}^{\pi/2} \sec x\, dx$ Notice: sec x Even function, so could do $\int_0^{\pi/2} \sec x\,dx$ and Double using symmetry

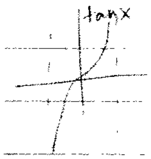
OR do it split $\int_{-\pi/2}^{0} \sec x\,dx + \int_0^{\pi/2} \sec x\,dx$

$= \lim_{s\to -\pi/2^+} \int_s^0 \sec x\,dx + \lim_{t\to \pi/2^-} \int_0^t \sec x\,dx$

$= \lim_{s\to -\pi/2^+} \ln|\sec x+\tan x| \Big|_s^0 + \lim_{t\to \pi/2^-} \ln|\sec x + \tan x| \Big|_0^t$

$= \lim_{s\to -\pi/2^+} \cancelto{0}{\ln|\sec \overset{1}{0} + \tan \overset{0}{0}|} \underbrace{- \ln|\sec \overset{\infty}{s} + \tan \overset{-\infty}{s}|}_{\text{Div.}} + \lim_{\cancel{t\to\pi/2^-}} \underbrace{\cancelto{\infty}{\ln|\sec \overset{\infty}{t} + \tan \overset{\infty}{t}|} - \cancelto{0}{\ln|\sec \overset{1}{0} + \tan \overset{0}{0}|}}_{\text{Div.}}$

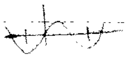




Div. $\frac{1}{\cos s} + \frac{\sin s}{\cos s}$

$\frac{0}{0}$ $\frac{1+\sin s}{\cos s} \overset{LH}{=} \frac{\cancelto{0}{\cos s}}{-\sin s}$ $\overset{0}{}$ $\boxed{\infty}$

~~ln|0|~~ → +∞



✓Diverges

certainly this piece Diverges; other first piece not as obvious, ~~but~~ only need one piece diverging

33. $\int_0^2 \frac{1}{(2x-1)^{2/3}}\,dx = \int_0^{1/2} \frac{1}{(2x-1)^{2/3}}\,dx + \int_{1/2}^{2} \frac{1}{(2x-1)^{2/3}}\,dx$

undef. at x = 1/2

$= \lim_{s\to 1/2^-} \int_0^s \frac{1}{(2x-1)^{2/3}}\,dx + \lim_{t\to 1/2^+} \int_t^2 \frac{1}{(2x-1)^{2/3}}\,dx$

or use ~~u-sub~~.

$= \lim_{s\to 1/2^-} \frac{1}{2}\cdot 3(2x-1)^{1/3}\Big|_0^s + \lim_{t\to 1/2^+} \frac{1}{2}\cdot 3(2x-1)^{1/3}\Big|_t^2$

$= \lim_{s\to 1/2^-} \cancelto{0}{\tfrac{3}{2}(2s-1)^{1/3}} - \tfrac{3}{2}\overset{-1}{(-1)^{1/3}} + \lim_{t\to 1/2^+} \tfrac{3}{2}(3)^{1/3} - \cancelto{0}{\tfrac{3}{2}(2t-1)^{1/3}}$

$= \frac{3}{2} + \frac{3}{2}(3)^{1/3} = \boxed{\frac{3}{2}\left[1+\sqrt[3]{3}\right]}$ ✓ Converges

34. $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{t\to 0^-} \int_t^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{t\to 0^-} 2\int_{\sqrt{t}}^1 e^u\,du = \lim_{t\to 0^-} 2e^u \Big|_{\sqrt{t}}^1$

$u = \sqrt{x}$
$du = \frac{1}{2\sqrt{x}}dx$
$2du = \frac{1}{\sqrt{x}}dx$

$x = t \Rightarrow u = \sqrt{t}$
$x = 1 \Rightarrow u = 1$

$= \lim_{t\to 0^-} 2e^1 - 2\cancelto{1}{e^{\sqrt{t}}} = \boxed{2(e^1-1)}$ ✓ converges

35. $\int_1^{\infty}\frac{\ln x}{x}dx = \lim\limits_{t\to\infty}\int_1^t \frac{\ln x}{x}dx = \lim\limits_{t\to\infty}\int_0^{\ln t} u\,du = \lim\limits_{t\to\infty}\frac{u^2}{2}\Big|_0^{\ln t} = \lim\limits_{t\to\infty}\frac{(\overset{\infty}{\cancel{\ln t}})^2}{2} - 0$ ✓

$u = \ln x$	$x = 1 \Rightarrow u = 0$
$du = \frac{1}{x}dx$	$x = t \Rightarrow u = \ln t$

$= \boxed{\infty}$ Diverges

36. $\int_0^{\infty}\frac{1}{x+x^2}dx = \int_0^1 \frac{1}{x+x^2}dx + \int_1^{\infty}\frac{1}{x+x^2}dx = \lim\limits_{s\to0^-}\int_s^1 \frac{1}{x(1+x)}dx + \lim\limits_{t\to\infty}\int_1^t \frac{1}{x(1+x)}dx$

Must Split Up

Decompose - Partial Fractions

$\left[\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}\right] x(x+1)$

$1 = A(x+1) + B(x)$

$= (A+B)x + A$

$\Rightarrow$ • $A = 1$

• $A+B = 0 \Rightarrow B = -1$

$= \lim\limits_{s\to0^-}\int_s^1 \frac{1}{x} - \frac{1}{x+1}dx \quad + \lim\limits_{t\to\infty}\int_1^t \frac{1}{x} - \frac{1}{1+x}dx$

$= \lim\limits_{s\to0^-} \ln|x| - \ln|x+1|\Big|_s^1 \quad + \lim\limits_{t\to\infty} \ln|x| - \ln|x+1|\Big|_1^t$

$= \lim\limits_{s\to0^-}(\overset{0}{\cancel{\ln 1}} - \ln 2) - \underbrace{(\ln|s| - \ln|s+1|)}_{\ln\left|\frac{\overset{0}{\cancel{s}}}{s+1}\right|} \quad + \lim\limits_{t\to\infty}\underbrace{(\ln|t| - \ln|t+1|)}_{\cancel{\ln}\left|\frac{\cancel{t}}{t+1}\right| \to 0} - (\overset{0}{\cancel{\ln 1}} - \ln 2)$

$\ln 0^{+}$

$-(-\infty)$

$\underbrace{-\cancel{\ln 2} \quad +\infty}_{1^{st}\text{ piece Diverges}} \quad + \cancel{\ln 2} = \boxed{+\infty}$

Diverges.

37. $\int_{-\infty}^{\infty}\frac{x}{(x^2+4)^{3/2}}dx = \int_{-\infty}^{0}\frac{x}{(x^2+4)^{3/2}}dx + \int_0^{\infty}\frac{x}{(x^2+4)^{3/2}}dx$

$= \lim\limits_{s\to-\infty}\int_s^0 \frac{x}{(x^2+4)^{3/2}}dx \quad + \lim\limits_{t\to\infty}\int_0^t \frac{x}{(x^2+4)^{3/2}}dx$

$x = s \Rightarrow u = s^2+4$	$u = x^2+4$	$x = 0 \Rightarrow u = 4$
$x = 0 \Rightarrow u = 4$	$du = 2x\,dx$	$x = t \Rightarrow u = t^2+4$
	$\frac{1}{2}du = x\,dx$	

$= \lim\limits_{s\to-\infty}\frac{1}{2}\int_{s^2+4}^{4}\frac{1}{u^{3/2}}du$

$= \lim\limits_{s\to-\infty}\frac{1}{2}\cdot(-2u^{-1/2})\Big|_{s^2+4}^{4}$

$= \lim\limits_{s\to-\infty}\frac{-1}{\sqrt{4}} - \left(\overset{0}{\cancel{-\frac{1}{\sqrt{s^2+4}}}}\right)$

$= -\frac{1}{2}$

$+ \lim\limits_{t\to\infty}\frac{1}{2}\int_4^{t^2+4}\frac{1}{u^{3/2}}du$

$+ \lim\limits_{t\to\infty}\frac{1}{2}\cdot(-2u^{-1/2})\Big|_4^{t^2+4}$

$+ \lim\limits_{t\to\infty}\overset{0}{\cancel{\frac{-1}{\sqrt{t^2+4}}}} - \left(-\frac{1}{\sqrt{4}}\right)$ ✓

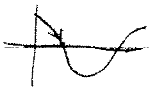
$= -\frac{1}{2} + \frac{1}{2} = \boxed{0}$ ✓ Converges

38. $\int_{-4}^{4}\frac{1}{(x+4)^{2/3}}dx = \lim\limits_{t\to-4^+}\int_t^4 \frac{1}{(x+4)^{2/3}} = \lim\limits_{t\to-4^+}3(x+4)^{1/3}\Big|_t^4 = \lim\limits_{t\to-4^+} 3\sqrt[3]{8} - \overset{0}{\cancel{3\sqrt[3]{0}}} = 3\cdot2 = \boxed{6}$ ✓

converges

39. $\int_0^{\pi/2} \frac{\sin x}{(\cos x)^{4/3}}\,dx = \lim_{t \to \pi/2^-} \int_0^t \frac{\sin x}{(\cos x)^{4/3}}\,dx = \lim_{t\to\pi/2^-} -\int_1^{\cos t} \frac{1}{u^{4/3}}\,du = \lim_{t\to\pi/2^-} -(-3)u^{-1/3}\Big|_1^{\cos t}$

$\boxed{\begin{array}{l} u=\cos x \\ du=-\sin x\,dx \\ -du=\sin x\,dx \end{array}}$ $\boxed{\begin{array}{l} x=0 \Rightarrow u=\cos 0=1 \\ x=t \Rightarrow u=\cos t \end{array}}$



$= \lim_{t\to\pi/2^-} \frac{3}{\underbrace{(\cos t)^{1/3}}_{0^+}} - \frac{3}{\cancel{(1)^{1/3}}\to 1} = \infty - 3 = \boxed{\infty}$ Diverges ✓

40. $\int_{-\infty}^{\infty} |x|e^{-x^2}dx = \int_{-\infty}^{0} |x|e^{-x^2}dx + \int_0^\infty |x|e^{-x^2}dx$

$= \int_{-\infty}^{0}(-x)e^{-x^2}dx + \int_0^\infty xe^{-x^2}dx$

$= \lim_{s\to-\infty} -\int_s^0 xe^{-x^2}dx + \lim_{t\to\infty}\int_0^t xe^{-x^2}dx$

$= \lim_{s\to-\infty} \frac{e^{-x^2}}{2}\Big|_s^0 + \lim_{t\to\infty} \frac{-e^{-x^2}}{2}\Big|_0^t$

$= \lim_{s\to-\infty} \frac{e^{0}\to 1}{2} - \cancel{\frac{e^{-s^2}}{2}}\to 0 + \lim_{t\to\infty} \cancel{-\frac{e^{-t^2}}{2}}\to 0 - \left(-\frac{e^{0}\to 1}{2}\right)$

$= \frac{1}{2} + \frac{1}{2} = \boxed{1}$ ✓ converges

41. $\int \frac{2x-5}{\underbrace{x^2+2x+2}}\,dx = \int \frac{2x+2-7}{x^2+2x+2}\,dx = \int \frac{2x+2}{x^2+2x+2}\,dx - 7\int \frac{1}{x^2+2x+2}\,dx$ ← complete the square

quadi.irred. already

$\boxed{\begin{array}{l} u=x^2+2x+2 \\ du=2x+2\,dx \end{array}}$ $-7\int \frac{1}{(x+1)^2+1}\,dx$

$= \int \frac{1}{u}\,du$ $\boxed{\begin{array}{l} w=x+1 \\ dw=dx \end{array}}$ $-7\int \frac{1}{w^2+1}\,dw$

$= \ln|u|$ $-7\arctan w$ $+C$

$= \boxed{\ln|x^2+2x+2| - 7\arctan(x+1) + C}$ ✓

42. $\int_0^1 \frac{3x^2-1}{x^3-x}\,dx = \int_0^{1/2} \frac{3x^2-1}{x^3-x}\,dx + \int_{1/2}^1 \frac{3x^2-1}{x^3-x}\,dx = \lim_{s\to 0^+}\int_s^{1/2} \frac{3x^2-1}{x^3-x}\,dx + \lim_{t\to 1^-}\int_{1/2}^t \frac{3x^2-1}{x^3-x}\,dx$

Must Split Up

$u = x^3-x$

$du = 3x^2-1\,dx$

$= \lim_{s\to0^+}\int_{s^3-s}^{-3/8} \frac{1}{u}\,du + \lim_{t\to1^-}\int_{-3/8}^{t^3-t} \frac{1}{u}\,du$

$x = s \Rightarrow u = s^3-s$

$x = \frac{1}{2} \Rightarrow u = \frac{1}{8} - \frac{1}{2} = \frac{1}{8} - \frac{4}{8} = -\frac{3}{8}$

$= \lim_{s\to0^+} \ln|u|\Big|_{s^3-s}^{-3/8} + \lim_{t\to1^-} \ln|u|\Big|_{-3/8}^{t^3-t} = \lim_{s\to0^+} \underbrace{\ln 3/8 - \cancel{\ln|s^3-s|}\to -\infty}_{+\infty} + \lim_{t\to1^-} \underbrace{\cancel{\ln|t^3-t|}\to -\infty - \ln 3/8}_{-\infty}$

Both Di Diverge(s)

43. $$\int_0^1 \frac{1}{e^x-e^{-x}}\,dx = \lim_{t\to0^+}\int_t^1 \frac{1}{e^x-1/e^x}\overset{\substack{\text{simplify}\\ \downarrow}}{\left(\frac{e^x}{e^x}\right)}dx = \lim_{t\to0^+}\int_t^1 \frac{e^x}{e^{2x}-1}\,dx = \lim_{t\to0^+}\int_{e^t}^{e} \frac{1}{u^2-1}\,du$$

$$\boxed{\begin{array}{c}u=e^x\\ du=e^x dx\end{array}} \quad \boxed{\begin{array}{l}x=t\Rightarrow u=e^t\\ x=1\Rightarrow u=e\end{array}}$$

$$\frac{1}{u^2-1} = \left(\frac{1}{(u-1)(u+1)} = \frac{A}{u-1}+\frac{B}{u+1}\right)(u-1)(u+1)$$

$1 = A(u+1)+B(u-1)$

$1 = (A+B)u + A - B$

- $A+B=0 \quad A=-B$
- $A-B=1 \quad -2B=1$

$B=-1/2 \Rightarrow A=1/2$

$$=\lim_{t\to0^+}\int_{e^t}^{e} \frac{\frac12}{u-1}-\frac{\frac12}{u+1}\,du$$

$$=\lim_{t\to0^+} \tfrac12\ln|u-1| - \tfrac12\ln|u+1|\Big|_{e^t}^{e}$$

$$=\lim_{t\to0^+}\frac12 \ln\left|\frac{u-1}{u+1}\right|\Bigg|_{e^t}^{e}$$

$$=\lim_{t\to0^+}\frac12\left[\ln\left|\frac{e-1}{e+1}\right| - \overset{-\infty}{\cancel{\ln\left|\frac{\overset{0}{e^t-1}}{e^t+1}\right|}}\right] = \boxed{+\infty}$$ ✓ Diverges

44. $$\int_0^1 \frac{e^x}{\sqrt{e^x-1}}\,dx = \lim_{t\to0^+}\int_t^1 \frac{e^x}{\sqrt{e^x-1}}\,dx = \lim_{t\to0^+}\int_{e^t-1}^{e-1}\frac{1}{\sqrt{u}}\,du = \lim_{t\to0^+} 2\sqrt{u}\Big|_{e^t-1}^{e-1} = \lim_{t\to0^+} 2\sqrt{e-1} - \overset{0}{\cancel{2\sqrt{e^t-1}}} = \boxed{2\sqrt{e-1}}$$ ✓

Converges

$$\boxed{\begin{array}{c}u=e^x-1\\ du=e^x dx\end{array}} \quad \boxed{\begin{array}{l}x=t\Rightarrow u=e^t-1\\ x=1\Rightarrow u=e-1\end{array}}$$

45. $$\int\frac{1}{x^2+4x+5}\,dx = \int\frac{1}{(x^2+4x+4)+1}\,dx = \int\frac{1}{(x+2)^2+1}\,dx = \int\frac{1}{u^2+1}\,du = \arctan u + C$$

Complete the square

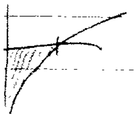
$$\boxed{\begin{array}{c}u=x+2\\ du=dx\end{array}}$$

$$=\boxed{\arctan(x+2)+C}$$ ✓

46. $$\int_0^\infty \sin^2 x\,dx = \lim_{t\to\infty}\int_0^t \frac{1-\cos(2x)}{2}\,dx = \lim_{t\to\infty}\frac12\int_0^t 1-\cos(2x)\,dx = \lim_{t\to\infty}\frac12\left[x-\frac12\sin(2x)\right]\Big|_0^t$$

$$=\lim_{t\to\infty}\frac12\left[t-\tfrac12\underbrace{\sin(2t)}_{\substack{\text{diverges}\\ \text{by oscillation}}}\right] - \tfrac12\left[0-\tfrac12\overset{0}{\cancel{\sin 0}}\right]$$ Diverges.

47. $$\int_0^1 \ln x\,dx = \lim_{t\to0^+}\int_t^1 \ln x\,dx \underset{\substack{\uparrow\\ \text{I.O.P.}}}{=} \lim_{t\to0^+} x\ln x - x\Big|_t^1 = \lim_{t\to0^+}(\overset{0}{\cancel{\ln 1}}-1) - (\overset{\substack{\text{L'H}\\ 0}}{\cancel{t\ln t}} - \overset{0}{\cancel{t}}) = \boxed{-1}$$ ✓ Converges



$$\lim_{\substack{t\to0^+\\ }} \underset{0\cdot(-\infty)}{t\ln t} = \lim_{t\to0^+}\frac{\ln t}{1/t} \overset{\text{L'H}}{=} \lim_{t\to0^+}\frac{1/t}{-1/t^2} = \lim_{t\to0^+} -t = 0$$

$$48.\ \int \frac{2x^2+3}{x(x-1)^2}\,dx = \int \frac{3}{x} - \frac{1}{x-1} + \underset{\uparrow}{\frac{5}{(x-1)^2}}\,dx = \boxed{3\ln|x| - \ln|x-1| - \frac{5}{x-1} + C} \checkmark$$

power rule
with sub.

Decompose - Partial Fractions

$$\left(\frac{2x^2+3}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}\right) x(x-1)^2$$

$$\Rightarrow 2x^2+3 = A(x-1)^2 + Bx(x-1) + Cx$$

$$= Ax^2 - 2Ax + A + Bx^2 - Bx + Cx$$

$$= (A+B)x^2 + (-2A-B+C)x + A$$

$\Rightarrow$ • $A+B=2 \quad B=2-A$

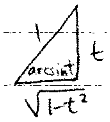
• $-2A-B+C=0 \quad -2(3)-(-1)+C=0 \quad C=+6-1=5$

• $A=3 \quad \Rightarrow B=2-3=-1$

$$49.\ \int_0^1 \frac{1}{(1-x^2)^{3/2}}\,dx = \lim_{t\to 1^-}\int_0^t \frac{1}{(1-x^2)^{3/2}}\,dx = \lim_{t\to 1^-}\int_0^{\arcsin t} \frac{1}{(\underbrace{1-\sin^2\theta}_{\cos^2\theta})^{3/2}}\cos\theta\, d\theta = \lim_{t\to 1^-}\int_0^{\arcsin t} \frac{\cos\theta}{\cos^3\theta}\,d\theta$$

$$\boxed{\begin{array}{l} x=\sin\theta \\ dx=\cos\theta\, d\theta \end{array}} \boxed{\begin{array}{l} x=0 \Rightarrow \theta=\arcsin 0=0 \\ x=t \Rightarrow \theta=\arcsin t \end{array}}$$

$$= \lim_{t\to 1^-}\int_0^{\arcsin t} \frac{1}{\cos^2\theta}\,d\theta = \lim_{t\to 1^-}\int_0^{\arcsin t} \sec^2\theta\, d\theta = \lim_{t\to 1^-} \tan\theta \Big|_0^{\arcsin t}$$



$$= \lim_{t\to 1^-} \tan(\arcsin t) - \cancelto{0}{\tan 0} = \lim_{t\to 1^-} \frac{t}{\sqrt{1-t^2}} = \frac{1}{0^+} = \boxed{+\infty} \checkmark$$

Diverges

$$50.\ \int_1^5 \frac{x}{\sqrt{x-1}}\,dx = \lim_{t\to 1^+}\int_t^5 \frac{x}{\sqrt{x-1}}\,dx = \lim_{t\to 1^+}\int_{t-1}^4 \frac{u+1}{\sqrt{u}}\,du = \lim_{t\to 1^+}\int_{t-1}^4 \sqrt{u} + u^{-1/2}\,du$$

$$\boxed{\begin{array}{l} u=x-1 \Rightarrow x=u+1 \\ du=dx \end{array}} \boxed{\begin{array}{l} x=t \Rightarrow u=t-1 \\ x=5 \Rightarrow u=4 \end{array}} = \lim_{t\to 1^+} \frac{2}{3}u^{3/2} + 2u^{1/2}\Big|_{t-1}^4$$

$$= \lim_{t\to 1^+}\left(\frac{2}{3}(4)^{3/2} + 2\sqrt{4}\right) - \left(\frac{2}{3}\cancelto{0}{(t-1)}^{3/2} + 2\cancelto{0}{\sqrt{t-1}}\right)$$

$$= \frac{2}{3}\cdot 8 + 4 = \frac{16}{3} + \frac{12}{3} = \boxed{\frac{28}{3}} \checkmark \text{ Converges}$$

51. $$\int_1^\infty \frac{1}{x(x^2+1)}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x(x^2+1)}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x}\ \frac{-x}{x^2+1}\,dx$$

Decompose-Partial Fractions

$$\left(\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}\right)x(x^2+1)$$

$$\Rightarrow 1 = A(x^2+1) + (Bx+C)x$$
$$= Ax^2 + A + Bx^2 + Cx$$
$$= (A+B)x^2 + Cx + A$$

$\Rightarrow$ • $A+B=0 \Rightarrow B=-A=-1$

• $C=0$

• $A=1$

$$= \lim_{t\to\infty} \ln x - \frac{1}{2}\ln(x^2+1)\Big|_1^t$$

$$= \lim_{t\to\infty}\left(\ln t - \ln\sqrt{t^2+1}\right) - \left(\overset{0}{\cancel{\ln 1}} - \frac{1}{2}\ln 2\right)$$

$$= \lim_{t\to\infty} \ln\frac{t}{\sqrt{t^2+1}} + \frac{1}{2}\ln 2$$

$$= \overset{0}{\cancel{\lim_{t\to\infty}\ln}}\sqrt{\frac{t^2 \to 1}{t^2+1}} + \frac{1}{2}\ln 2 = \boxed{\frac{1}{2}\ln 2}$$ ✓ converges

52. $$\int_{-\infty}^\infty x\sin x\,dx = \int_{-\infty}^0 x\sin x\,dx + \int_0^\infty x\sin x\,dx = \underbrace{\lim_{s\to-\infty}\int_s^0 x\sin x\,dx}_{(**)} + \underbrace{\lim_{t\to\infty}\int_0^t x\sin x\,dx}_{(*)}$$

$$\boxed{\begin{matrix} u=x & dv=\sin x\,dx \\ du=dx & v=-\cos x\end{matrix}}$$

$$(*)\ \lim_{t\to\infty}\int_0^t x\sin x\,dx = \lim_{t\to\infty} -x\cos x\Big|_0^t - \int_0^t \cos x\,dx = \lim_{t\to\infty} -x\cos x - \sin x\Big|_0^t$$

$$= \lim_{t\to\infty}\underbrace{\left(-t\cos t - \sin t\right)}_{\text{Div. by oscillation}} - \left(\overset{0}{\cancel{0\cos 0}} - \overset{0}{\cancel{\sin 0}}\right)$$

✓ Diverges - Done

Don't need other piece, but

$$(**)\ \lim_{s\to-\infty} -x\cos x - \sin x\Big|_s^0 = \lim_{s\to-\infty}\left(\overset{0}{\cancel{-0\cos 0}} - \overset{0}{\cancel{\sin 0}}\right) - \underbrace{\left(-s\cos s - \sin s\right)}_{\text{Div. by oscillation}}$$

53. $$\int_{-\infty}^\infty \frac{1}{x^2-6x+10}\,dx = \int_{-\infty}^0 \frac{1}{x^2-6x+10}\,dx + \int_0^\infty \frac{1}{x^2-6x+10}\,dx = \lim_{s\to-\infty}\int_s^0 \frac{1}{(x-3)^2+1}\,dx + \lim_{t\to\infty}\int_0^t \frac{1}{(x-3)^2+1}\,dx$$

$$\boxed{\begin{matrix} u=x-3 \\ du=dx\end{matrix}}\ \boxed{\begin{matrix} x=s \Rightarrow u=s-3 \\ x=0 \Rightarrow u=-3\end{matrix}}\ \boxed{\begin{matrix} x=0 \Rightarrow u=-3 \\ x=t \Rightarrow u=t-3\end{matrix}}$$

$$= \lim_{s\to-\infty}\int_{s-3}^{-3}\frac{1}{u^2+1}\,du + \lim_{t\to\infty}\int_{-3}^{t-3}\frac{1}{u^2+1}\,du = \lim_{s\to-\infty}\arctan u\Big|_{s-3}^{-3} + \lim_{t\to\infty}\arctan u\Big|_{-3}^{t-3}$$

$$= \lim_{s\to-\infty}\cancel{\arctan(-3)} - \overset{-\pi/2}{\arctan(s-3)} + \lim_{t\to\infty}\overset{\pi/2}{\arctan(t-3)} - \cancel{\arctan(-3)}$$

$$= \pi/2 + \pi/2 = \boxed{\pi}$$ ✓ converges

54. $$\int_{-\infty}^\infty x\,dx = \int_{-\infty}^0 x\,dx + \int_0^\infty x\,dx = \lim_{s\to-\infty}\int_s^0 x\,dx + \lim_{t\to\infty}\int_0^t x\,dx = \lim_{s\to-\infty}\frac{x^2}{2}\Big|_s^0 + \lim_{t\to\infty}\frac{x^2}{2}\Big|_0^t$$

$$= \underbrace{\lim_{s\to-\infty} 0 - s^2/2} \qquad + \underbrace{\lim_{t\to\infty}\cancel{\frac{t^2}{2}}^{\infty} - 0}$$ $\Rightarrow$ Diverges ✓

55. $\int \frac{x^4-x^3-x-1}{x^3-x^2}\,dx = \int x - \frac{x+1}{x^3-x^2}\,dx = \int x - \frac{x+1}{x^2(x-1)}\,dx = \int x - \left(\frac{-2}{x} \frac{-1}{x^2} + \frac{2}{x-1}\right) dx$

$= \int x + \frac{2}{x} + \frac{1}{x^2} - \frac{2}{x-1}$

$= \boxed{\frac{x^2}{2} + 2\ln|x| - \frac{1}{x} - 2\ln|x-1| + C}$ ✓

$$\begin{array}{r} x \\ x^3-x^2\,\big)\overline{x^4-x^3-x-1} \\ \underline{-(x^4-x^3)} \\ -x-1 \end{array}$$

Decompose - Partial Fractions

$\left(\frac{x+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}\right) x^2(x-1)$

$\Rightarrow \quad x+1 = A\,x(x-1) + B(x-1) + Cx^2$

$= Ax^2 - Ax + Bx - B + Cx^2$

$= (A+C)x^2 + (-A+B)x - B$

$\Rightarrow$ • $A+C=0 \quad C=-A=2$

• $(-A+B)=1 \quad A=B-1=-1-1=-2$

• $-B=1 \Rightarrow B=-1$

56. $\int_0^\infty \frac{x}{e^x}\,dx = \lim_{t\to\infty} \int_0^t xe^{-x}\,dx = \lim_{t\to\infty} -xe^{-x}\Big|_0^t - \int_0^t -e^{-x}\,dx = \lim_{t\to\infty} -xe^{-x} - e^{-x}\Big|_0^t = \lim_{t\to\infty} \frac{-x}{e^x} - \frac{1}{e^x}\Big|_0^t$

$$\boxed{\begin{array}{ll} u=x & dv=e^{-x}dx \\ du=dx & v=-e^{-x} \end{array}}$$

$= \lim_{t\to\infty}\left(\cancelto{0}{\frac{-t}{e^t}} - \cancelto{0}{\frac{1}{e^t}}\right) - \left(\cancelto{0}{\frac{0}{1}} - \frac{1}{1}\right)$

$\lim_{t\to\infty} \frac{t}{e^t} \overset{\infty/\infty}{\underset{L'H}{=}} \lim_{t\to\infty} \cancelto{0}{\frac{1}{e^t}} = 0$

$= \boxed{1}$ ✓ Converges

57. $\int_{-5}^0 \frac{x}{x^2+4x-5}\,dx = \int_{-5}^0 \frac{x}{(x+5)(x-1)}\,dx = \lim_{t\to -5^+} \int_t^0 \frac{x}{(x+5)(x-1)}\,dx = \lim_{t\to -5^+} \int_t^0 \frac{\frac{5}{6}}{x+5} + \frac{1/6}{x-1}\,dx$

Decompose - Partial Fractions

$\left(\frac{x}{(x+5)(x-1)} = \frac{A}{x+5} + \frac{B}{x-1}\right)(x+5)(x-1)$

$\Rightarrow \quad x = A(x-1) + B(x+5)$

$= (A+B)x + (-A+5B)$

$\Rightarrow$ • $A+B=1 \Rightarrow B=1-A$

• $-A+5B=0 \quad -A+5(1-A)=0$

or add $\quad 6B=1 \qquad -6A+5=0$

$B=1/6 \qquad A=+5/6$

$\Rightarrow B=1-5/6=1/6.$

$= \lim_{t\to -5^+} 5/6 \ln|x+5| + 1/6 \ln|x-1| \Big|_t^0$

$= \lim_{t\to -5^+} \left(5/6 \ln|5| + \cancelto{0}{1/6 \ln 1}\right) - \left(\cancelto{-\infty}{5/6 \ln|\underbrace{t+5}_{0^+}|} + 1/6\,|t-1|\right)$

$= 5/6 \ln 5 - 5/6(-\infty) - 1/6 \ln|6|$

$= 5/6 \ln 5 + \infty - 1/6 \ln 6 = \boxed{+\infty}$ ✓ Diverges

58. $\int_{-5}^{0} \frac{1}{x^2+4x-5}\,dx = \lim_{t\to -5^+} \int_t^0 \frac{1}{(x+5)(x-1)}\,dx = \lim_{t\to -5^+} \int_t^0 \frac{-1/6}{x+5} + \frac{1/6}{x-1}\,dx$

Decompose - Partial Fractions

$\left(\frac{1}{(x+5)(x-1)} = \frac{A}{x+5} + \frac{B}{x-1}\right)(x+5)(x-1)$

$\Rightarrow \quad 1 = A(x-1) + B(x+5)$

$= (A+B)x + (-A+5B)$

$\Rightarrow \cdot\ A+B=0 \Rightarrow B=-A$

$\cdot\ -A+5B=1 \quad -A+5(-A)=1$

$-6A=1$

$A=-1/6$

$\Rightarrow B=1/6$

$= \lim_{t\to -5^+} -\frac16 \ln|x+5| + \frac16 \ln|x-1| \Big|_t^0$

$= \lim_{t\to -5^+} \left(-\frac16 \ln 5 + \overset{0}{\cancel{\tfrac16 \ln 1}}\right) - \left(-\frac16 \ln|\overset{0^+}{t+5}| + \frac16 \ln|t-1|\right)$

$= -\frac16 \ln 5 + \frac16(+\infty) - \frac16 \ln 6 = \boxed{+\infty}$ ✓ Diverges.

59. $\int_{-1}^{1} \frac{1}{x^3}\,dx = \int_{-1}^{0} \frac{1}{x^3}\,dx + \int_0^1 \frac{1}{x^3}\,dx = \lim_{s\to 0^+} \int_{-1}^{s} x^{-3}\,dx + \lim_{t\to 0^-} \int_t^1 x^{-3}\,dx$

$= \lim_{s\to 0^+} \frac{x^{-2}}{-2}\Big|_{-1}^{s} + \lim_{t\to 0^-} \frac{x^{-2}}{-2}\Big|_t^1$

$= \lim_{s\to 0^+} \overset{\infty}{\frac{-1}{2s^2}} - \left(\frac{-1}{2(-1)^2}\right) + \lim_{t\to 0^-} \frac{-1}{2} - \left(\frac{-1}{2t^2}\right)$

$= \underbrace{-\infty + 1/2}_{\text{Diverges}} - 1/2 \underbrace{+\infty}_{\text{Diverges}} = \text{Diverges}$ ✓

60. $\int \frac{x^5+2}{x^2-1}\,dx = \int x^3+x + \frac{x+2}{x^2-1}\,dx = \int x^3+x + \frac{3/2}{x-1} - \frac{1/2}{x+1}\,dx$

$$\begin{array}{r} x^3+x \\ x^2-1 \overline{)\ x^5+2} \\ -(x^5-x^3) \\ \hline x^3+2 \\ -(x^3-x) \\ \hline x+2 \end{array}$$

$\boxed{= \frac{x^4}{4} + \frac{x^2}{2} + \frac32 \ln|x-1| - \frac12 \ln|x+1| + C}$ ✓

Decompose - Partial Fractions

$\frac{x+2}{x^2-1}\ \left(\frac{x+2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}\right)(x-1)(x+1)$

$\Rightarrow x+2 = A(x+1) + B(x-1)$

$= (A+B)x + A-B$

$\Rightarrow \cdot\ A+B=1 \Rightarrow B=1-A$

$\cdot\ A-B=2 \quad A-(1-A)=2$

$A-1+A=2$

$2A=3$

$$61.\ \int_0^6 \frac{1}{(x-3)^2}\,dx = \int_0^2 \frac{1}{(x-2)^2}\,dx + \int_2^6 \frac{1}{(x-2)^2}\,dx = \lim_{s\to 2^-}\int_0^s \frac{1}{(x-2)^2}\,dx + \lim_{t\to 2^+}\int_t^6 \frac{1}{(x-2)^2}\,dx$$

$$= \lim_{s\to 2^-} \frac{-1}{x-2}\Big|_0^s + \lim_{t\to 2^+} \frac{-1}{x-2}\Big|_t^6$$

$$= \lim_{s\to 2^-} \frac{-1}{\underbrace{s-2}_{0^-}} - \left(\frac{-1}{-2}\right) + \lim_{t\to 2^+} \frac{-1}{4} - \left(\frac{-1}{t-2}\right)$$

$$+\infty - 1/2 - 1/4 \quad +\infty = \boxed{\infty}$$ ✓ Diverges.

$$62.\ \int_0^\infty \frac{1}{x^2+3x+2}\,dx = \lim_{t\to\infty}\int_0^t \frac{1}{(x+2)(x+1)}\,dx = \lim_{t\to\infty}\int_0^t \frac{-1}{x+2} + \frac{1}{x+1}\,dx$$

Decompose - Partial Fractions

$$\left(\frac{1}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}\right)(x+2)(x+1)$$

$$\Rightarrow\ 1 = A(x+1) + B(x+2)$$

$$= (A+B)x + A + 2B$$

$$\Rightarrow \bullet\ A+B=0 \ \Rightarrow B=-A$$

$$\bullet\ A+2B=1 \quad A+2(-A)=1$$

$$-A=1$$

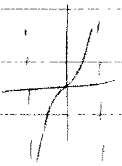
$$A=-1$$

$$\Rightarrow B=1$$

$$= \lim_{t\to\infty} -\ln|x+2| + \ln|x+1|\Big|_0^t$$

$$= \lim_{t\to\infty} -\ln|t+2| + \ln|t+1| - (-\ln 2 + \overset{0}{\cancel{\ln 1}})$$

$$= \lim_{t\to\infty} \overset{0}{\cancel{\ln\left|\frac{t+1}{t+2}\right|}} + \ln 2 = \boxed{\ln 2}$$ ✓ converges.



$$63.\ \int_0^{\pi/2} \tan^2 x\,dx = \lim_{t\to \pi/2^-}\int_0^t \sec^2 x - 1\,dx = \lim_{t\to\pi/2^-} \tan x - x\Big|_0^t = \lim_{t\to\pi/2^-} (\overset{+\infty}{\cancel{\tan t}} - t) - (\overset{0}{\cancel{\tan 0}} - 0) = \boxed{+\infty}$$ ✓

Diverges

$$64.\ \int_0^2 \frac{1}{(4-x^2)^{3/2}}\,dx = \lim_{t\to 2^-}\int_0^t \frac{1}{(4-x^2)^{3/2}}\,dx = \lim_{t\to 2^-}\int_0^{\arcsin t/2} \frac{1}{\underbrace{(4-4\sin^2\theta)^{3/2}}_{(4\cos^2\theta)^{3/2}}}\,2\cos\theta\,d\theta$$

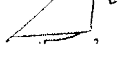
$\theta = \arcsin\frac{x}{2}$ ←

$$\boxed{\begin{array}{l} x = 2\sin\theta \\ dx = 2\cos\theta\,d\theta \end{array}} \quad \boxed{\begin{array}{l} x=0 \Rightarrow \theta = \arcsin 0 = 0 \\ x=t \Rightarrow \theta = \arcsin t/2 \end{array}}$$

$$= \lim_{t\to 2^-}\int_0^{\arcsin t/2} \frac{2}{4^{3/2}}\cdot\frac{1}{\cos^2\theta}\,d\theta = \lim_{t\to 2^-}\frac{1}{4}\int_0^{\arcsin t/2} \sec^2\theta\,d\theta = \lim_{t\to 2^-}\frac{1}{4}\tan\theta\Big|_0^{\arcsin t/2}$$

$$= \lim_{t\to 2^-} \frac{1}{4}\overset{+\infty}{\overset{\tan(\pi/2)}{\cancel{\tan(\underset{\arcsin 1}{\cancel{\arcsin t/2}})}}} - 1/4\,\overset{0}{\cancel{\tan 0}} = \boxed{+\infty}$$ ✓ Diverges.

or go back to



$$\lim_{t\to 2^-}\frac{1}{4}\frac{t}{\sqrt{4-t^2}} - 0 = \frac{1}{4}\frac{2}{0^+} = +\infty$$

$$65. \int_1^{32} \frac{1}{\sqrt[5]{x-32}}\,dx = \lim_{t\to 32^-}\int_1^t (x-32)^{-1/5}dx = \lim_{t\to 32^-}\int_{-31}^{t-32} u^{-1/5}du = \lim_{t\to 32^-}\frac{5}{4}u^{4/5}\Big|_{-31}^{t-32}$$

$$\boxed{\begin{matrix} u=x\,32 \\ du=dx \end{matrix}} \quad \boxed{\begin{matrix} x=1 \Rightarrow u=-31 \\ x=t \Rightarrow u=t-32 \end{matrix}}$$

$$=\lim_{t\to 32^-} \underbrace{\frac{5}{4}(t-32)^{4/5}}_{\to 0} - \frac{5}{4}(-31)^{4/5}$$

$$=\boxed{-\frac{5}{4}(-31)^{4/5}}$$ ✓ converges

$$66. \int_{-\infty}^1 x e^{4x}dx = \lim_{s\to-\infty}\int_s^1 x e^{4x}dx \overset{\text{I.B.P.}}{=} \lim_{s\to-\infty}\frac{1}{4}xe^{4x}\Big|_s^1 - \int_s^1 \frac{1}{4}e^{4x}dx$$

$$\boxed{\begin{matrix} u=x & dv=e^{4x}dx \\ du=dx & v=\frac{1}{4}e^{4x} \end{matrix}}$$

$$=\lim_{s\to-\infty}\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\Big|_s^1$$

$$=\lim_{s\to-\infty}\left(\frac{1}{4}e^4 - \frac{1}{16}e^4\right) - \left(\cancel{\frac{1}{4}se^{4s}}^{\text{L'H}\,0} - \cancel{\frac{1}{16}e^{4s}}^{0}\right)$$

$$\lim_{s\to-\infty} se^{4s} = \lim_{s\to-\infty}\frac{s}{e^{-4s}} \overset{\frac{-\infty}{\infty}\ \text{L'H}}{=} \lim_{s\to-\infty}\frac{1}{-4e^{-4s}} = \lim_{s\to-\infty} -\frac{1}{4}\cancel{e^{4s}}^{0} = 0$$

$$= \frac{1}{4}e^4 - \frac{1}{16}e^4 = \left(\frac{4}{16}-\frac{1}{16}\right)e^4 = \boxed{\frac{3}{16}e^4}$$ ✓ converges.

$$67. \int \frac{1}{(x+1)^2(x+2)}dx = \int \frac{1}{x+2} - \frac{1}{x+1} + \frac{1}{(x+1)^2}dx = \boxed{\ln|x+2| - \ln|x+1| - \frac{1}{x+1} + C}$$ ✓

Decompose- Partial Fractions

$$\left(\frac{1}{(x+2)(x+1)^2} = \frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\right)(x+2)(x+1)^2$$

$$1 = A(x+1)^2 + B(x+2)(x+1) + C(x+2)$$

$$= Ax^2 + \underline{2Ax} + A + Bx^2 + \underline{3Bx} + 2B + \underline{Cx} + 2C$$

$$= (A+B)x^2 + (2A+3B+C)x + A + 2B + 2C$$

$\Rightarrow$ • $A+B=0 \Rightarrow B=-A$

• $2A+3B+C=0$ $\quad 2A+3(-A)+C=0 \Rightarrow C=A$

• $A+2B+2C=1$ $\quad A+2(-A)+2(A)=1$

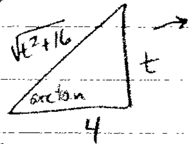
$A=1$

$\Rightarrow C=1$

$\Rightarrow B=-1$

$$\frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta} = \frac{\cos\theta}{\sin^2\theta} = \frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta}$$

$$\frac{\sec\theta}{\tan^2\theta} = \cot\theta\csc\theta$$



$$\to \csc\left(\arctan\tfrac{t}{4}\right) \overset{\frac{1}{\sin}}{} = \frac{\sqrt{t^2+16}}{t}$$

$$68. \int_0^1 \frac{1}{x^2\sqrt{x^2+16}}dx = \lim_{t\to0^+}\int_t^1 \frac{1}{x^2\sqrt{x^2+16}}dx = \lim_{t\to0^+}\int_{\arctan t/4}^{\arctan 1/4} \frac{1}{16\tan^2\theta\,\underbrace{\sqrt{16\tan^2\theta+16}}_{4\sec\theta}}\cancel{4}\sec^2\theta\, d\theta = \lim_{t\to0^+}\frac{1}{16}\int_{\arctan t/4}^{\arctan t/4}\frac{\sec\theta}{\tan^2\theta}d\theta$$

CHALLENGE

$$\boxed{\begin{matrix} x=4\tan\theta \\ dx=4\sec^2\theta\, d\theta \end{matrix}} \quad \boxed{\begin{matrix} x=t \Rightarrow \theta=\arctan t/4 \\ x=1 \Rightarrow \theta=\arctan 1/4 \end{matrix}}$$

$$=\lim_{t\to0^+}\frac{1}{16}\int_{\arctan t/4}^{\arctan t/4}\cot\theta\csc\theta\, d\theta = \lim_{t\to0^+}\frac{1}{16}(-\csc\theta)\Big|_{\arctan t/4}^{\arctan 1/4} = \lim_{t\to0^+}-\frac{1}{16}\left(\csc(\arctan 1/4) - \underbrace{\csc(\arctan t/4)}_{\frac{\sqrt{t^2+16}}{t}\ \to +\infty}\right) = \boxed{+\infty}$$ ✓

Diverges

$$\overset{\text{u-sub.}}{\downarrow} \qquad \overset{\text{arctan}}{\downarrow} \qquad \swarrow \frac{1}{4}\int \frac{1}{(\frac{x}{2})^2+1}\,dx$$

69. $\displaystyle\int \frac{4x^2+7x+6}{(x+2)(x^2+4)}\,dx = \int \frac{1}{x+2} + \frac{3x+1}{x^2+4}\,dx = \ln|x+2| + 3\int \frac{x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx$

$w = x/2$
$dw = \frac{1}{2}dx$
$2dw = dx$

$$= \boxed{\ln|x+2| + \frac{3}{2}\ln|x^2+4| + \frac{1}{2}\arctan\frac{x}{2} + C} \ ✓$$

Decompose - Partial Fractions

$$\left(\frac{4x^2+7x+6}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}\right)(x+2)(x^2+4)$$

$\Rightarrow 4x^2+7x+6 = A(x^2+4) + (Bx+C)(x+2)$

$= Ax^2 + 4A + Bx^2 + Cx + 2Bx + 2C$

$= (A+B)x^2 + (2B+C)x + 4A + 2C$

$\Rightarrow \cdot\ A+B=4 \Rightarrow B=4-A$

$\cdot\ 2B+C=7 \quad 2(4-A)+C=7 \Rightarrow 8-2A+C=7 \Rightarrow C=2A-1$

$\cdot\ 4A+2C=6$

$4A+2(2A-1)=6$
$8A-2=6$
$8A=8$
$\Rightarrow A=1$
$\Rightarrow B=4-1=3$
$\Rightarrow C=2-1=1$

70. $\displaystyle\int_1^\infty \frac{1}{x(x+1)}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x(x+1)}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x} - \frac{1}{x+1}\,dx$

$\displaystyle= \lim_{t\to\infty} \ln|x| - \ln|x+1| \Big|_1^t$

$\displaystyle= \lim_{t\to\infty} \Big(\underbrace{\ln|t| - \ln|t+1|}_{\overset{0}{\ln|\frac{t}{t+1}|}}\Big) - (\overset{0}{\cancel{\ln 1}} - \ln 2) = \boxed{\ln 2}$ ✓ converges

Decompose - Partial Fractions

$$\left(\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}\right)x(x+1)$$

$1 = A(x+1) + Bx$
$= (A+B)x + A$

$\Rightarrow \cdot\ A+B=0$
$\cdot\ A=1 \Rightarrow B=-1$

$\displaystyle\int \frac{1}{x} - \frac{1}{x+1}\,dx = \ln|x| - \ln|x+1|$
$\displaystyle= \ln\left|\frac{x}{x+1}\right|$

71. $\displaystyle\int_{-3}^{3} \frac{1}{x(x+1)}\,dx = \int_{-3}^{-1}\frac{1}{x(x+1)}\,dx + \int_{-1}^{-1/2}\frac{1}{x(x+1)}\,dx + \int_{-1/2}^{0}\frac{1}{x(x+1)}\,dx + \int_0^3 \frac{1}{x(x+1)}\,dx$

Recall - Decomposition - Partial Fractions

$\displaystyle\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$

$\displaystyle= \lim_{s\to-1^-}\int_{-3}^{s} \frac{1}{x} - \frac{1}{x+1}\,dx + \lim_{t\to-1^+}\int_t^{-1/2}\frac{1}{x} - \frac{1}{x+1}\,dx + \lim_{k\to0^-}\int_{-1/2}^{k}\frac{1}{x} - \frac{1}{x+1}\,dx + \lim_{m\to0^+}\int_m^3 \frac{1}{x} - \frac{1}{x+1}\,dx$

$\displaystyle= \lim_{s\to-1^-} \ln\left|\frac{x}{x+1}\right|\Big|_{-3}^{s} + \lim_{t\to-1^+}\ln\left|\frac{x}{x+1}\right|\Big|_t^{-1/2} + \lim_{k\to0^-}\ln\left|\frac{x}{x+1}\right|\Big|_{-1/2}^{k} + \lim_{m\to0^+}\ln\left|\frac{x}{x+1}\right|\Big|_m^3$

$\displaystyle= \lim_{s\to-1^-}\left(\overset{\infty}{\ln\left|\frac{s}{s+1}\right|} - \ln\left|\frac{-3}{-2}\right|\right) + \lim_{t\to-1^+}\left(\overset{0}{\ln\left|\frac{-1/2}{1/2}\right|} - \overset{\infty}{\ln\left|\frac{t}{t+1}\right|}\right) + \lim_{k\to0^-}\left(\overset{\infty}{\ln\left|\frac{k}{k+1}\right|} - \overset{0}{\ln\left|\frac{-1/2}{1/2}\right|}\right) + \lim_{m\to0^+}\left(\ln\left|\frac{3}{4}\right| - \overset{-(-\infty)}{\ln\left|\frac{m}{m+1}\right|}\right)$

Altogether
Diverges.

$$72.\ \int_{-3}^{1} \frac{1}{x^2-4}\,dx = \int_{-3}^{1} \frac{1}{(x-2)(x+2)}\,dx = \int_{-3}^{-2} \frac{1}{(x-2)(x+2)}\,dx + \int_{-2}^{1} \frac{1}{(x-2)(x+2)}\,dx$$

und at $x=-2$

Decompose - Partial Fractions

$$\left(\frac{1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}\right)(x-2)(x+2)$$

$$\Rightarrow 1 = A(x+2) + B(x-2)$$

$$= (A+B)x + 2A - 2B$$

$\Rightarrow \cdot\ A+B=0 \Rightarrow B=-A$

$\cdot\ 2A-2B=1 \quad 2A-2(-A)=1$

$4A=1$

$A=1/4.$

$\Rightarrow B=-1/4$

$$= \lim_{s\to-2^-} \int_{-3}^{s} \frac{1}{(x-2)(x+2)}\,dx + \lim_{t\to-2^+} \int_{t}^{1} \frac{1}{(x-2)(x+2)}\,dx \qquad \frac{\frac{1}{4}}{x-2} - \frac{1/4}{x+2}$$

$$= \lim_{s\to-2^-} \frac{1}{4}\int_{-3}^{s} \frac{1}{x-2} - \frac{1}{x+2}\,dx + \lim_{t\to-2^+} \frac{1}{4}\int_{t}^{1} \frac{1}{x-2} - \frac{1}{x+2}\,dx$$

$$= \lim_{s\to-2^-} \frac{1}{4} \ln\left|\frac{x-2}{x+2}\right|\Bigg|_{-3}^{s} + \lim_{t\to-2^+} \frac{1}{4}\left(\ln|x-2| - \ln|x+2|\right)\Bigg|_{t}^{1}$$

$$+ \lim_{t\to-2^+} \frac{1}{4} \ln\left|\frac{x-2}{x+2}\right|\Bigg|_{t}^{1}$$

$$= \lim_{s\to-2^-} \frac{1}{4}\left[\ln\left|\frac{s-2}{s+2}\right| - \ln\left|\frac{-5}{-1}\right|\right] \qquad \ln\left|\frac{-4}{0}\right| \to \infty$$

Diverges

$$+ \lim_{t\to-2^+} \frac{1}{4}\left[\ln\left|-\frac{1}{3}\right| - \ln\left|\frac{t-2}{t+2}\right|\right] \qquad -\ln\left|\frac{-4}{0}\right| \to \boxed{-\infty}$$

Diverges

Altogether Diverges

$$73.\ \int_0^1 \arcsin x\,dx = x\arcsin x\Big|_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx = x\arcsin x\Big|_0^1 - \lim_{t\to1^-}\int_0^t \frac{x}{\sqrt{1-x^2}}\,dx$$

$$\boxed{\begin{array}{ll} u = \arcsin x & dv = dx \\ du = \frac{1}{\sqrt{1-x^2}}\,dx & V = x \end{array}}$$

undef. at $x=1$

$u = 1-x^2 \qquad x=0 \Rightarrow u=1$

$du = -2x\,dx \qquad x=t \Rightarrow u=1-t^2$

$-1/2\,du = x\,dx$

$$= x\arcsin x\Big|_0^1 - \lim_{t\to1^-} -1/2 \int_1^{1-t^2} \frac{1}{\sqrt{u}}\,du \qquad u^{-1/2}$$

$$= x\arcsin x\Big|_0^1 + \lim_{t\to1^-} \frac{1}{\cancel{2}} \cdot \cancel{2}\sqrt{u}\,\Big|_1^{1-t^2}$$

$$= 1\overset{\pi/2}{\arcsin 1} - \overset{0}{0\arcsin 0} + \lim_{t\to1^-} \overset{0}{\sqrt{1-t^2}} - \sqrt{1}$$

$$= \boxed{\pi/2 - 1} \text{ converges.}$$

$$74.\ \int_0^\infty \cosh x\,dx = \lim_{t\to\infty}\int_0^t \cosh x\,dx = \lim_{t\to\infty} \sinh x\Big|_0^t = \lim_{t\to\infty} \overset{\infty}{\sinh t} - \overset{0}{\sinh 0} = \lim_{t\to\infty} \frac{\overset{\infty}{e^t} - \overset{0}{1/e^t}}{2} = \boxed{\infty} \text{ Diverges.}$$

$$75.\ \int \frac{2x^3}{x^2+3}\,dx = \int 2x - \frac{6x}{x^2+3}\,dx = \boxed{x^2 - 3\ln|x^2+3| + C}$$

u-sub.

$$\begin{array}{r} 2x \\ x^2+3\,\overline{)\,2x^3} \\ -(2x^3+6x) \\ \hline 0 - 6x \end{array}$$

76. $\int \frac{x^2-1}{x^2+1}\,dx = \int \frac{x^2+1-2}{x^2+1}\,dx = \int \cancelto{1}{\frac{x^2+1}{x^2+1}} \; \frac{-2}{x^2+1}\,dx = \boxed{x - 2\arctan x + C}$

or $x^2+1 \overset{1}{\overline{\big)\, x^2-1}}$ $\begin{array}{r} -(x^2+1) \\ \hline -2 \end{array}$ $\rightarrow \frac{x^2-1}{x^2+1} = 1 \; \frac{-2}{x^2+1} \nearrow$

77. $\int \frac{\cos x\,(\sin^3 x + 7\sin x + 1)}{\sin^2 x + 1}\,dx = \int \frac{u^3+7u+1}{u^2+1}\,du = \int u + \frac{6u+1}{u^2+1}\,du = \int u + \overset{\text{u-sub}}{\overset{\downarrow}{6\frac{u}{u^2+1}}} + \overset{\text{arctan}}{\overset{\downarrow}{\frac{1}{u^2+1}}}\,dx$

$u = \sin x$
$du = \cos x\,dx$

$u^2+1 \overset{u}{\overline{\big)\, u^3+7u+1}}$ $\begin{array}{r} -(u^3+u) \\ \hline 6u+1 \end{array}$

$= \frac{u^2}{2} + 3\ln|u^2+1| + \arctan u + C$

$= \boxed{\frac{\sin^2 x}{2} + 3\ln|\sin^2 x + 1| + \arctan(\sin x) + C}$

78. $\int \frac{x^2+5x+2}{(x+1)(x^2+1)}\,dx = \int \frac{-1}{x+1} + \frac{2x+3}{x^2+1}\,dx = \int \frac{-1}{x+1} + \frac{2x}{x^2+1} + \frac{3}{x^2+1}\,dx$

$= \boxed{-\ln|x+1| + \ln|x^2+1| + 3\arctan x + C}$

Decompose - Partial Fractions

$\left(\frac{x^2+5x+2}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}\right)(x+1)(x^2+1)$

$x^2+5x+2 = A(x^2+1) + (Bx+C)(x+1)$
$= Ax^2 + A + Bx^2 + Bx + Cx + C$
$= (A+B)x^2 + (B+C)x + A + C$

$\Rightarrow$ • $A+B = 1 \rightarrow B = 1-A$
• $B+C = 5$ $\quad (1-A)+(2-A) = 5$
• $A+C = 2 \rightarrow C = 2-A$

$3-2A = 5$
$2A = -2$
$A = -1$
$\Rightarrow B = 1-(-1) = 2$
$C = 2-(-1) = 3$