20. $f(x) = \dfrac{\sinh x}{x^2 + \cosh x + 3}$ $\quad f'(x) = \dfrac{(x^2+\cosh x+3)\cosh x - \sinh x\,(2x+\sinh x)}{[x^2+\cosh x+3]^2}$

21. $f(x) = \dfrac{\sinh(3x)}{\cosh(4x)}$ $\quad f'(x) = \dfrac{\cosh(4x)\cdot\cosh(3x)\cdot 3 - \sinh(3x)\sinh(4x)\cdot 4}{[\cosh(4x)]^2}$

22. $f(x) = \cosh^{-1}(3x+4)$ $\quad f'(x) = \dfrac{1}{\sqrt{(3x+4)^2-1}}\cdot 3$

23. $f(x) = \dfrac{\arctan(x+2)}{\sec^2 x}$ $\quad f'(x) = \dfrac{\sec^2 x\cdot\frac{1}{(x+2)^2+1} - \arctan(x+2)\cdot 2\sec x\cdot \sec x \tan x}{\sec^4 x}$

24. $f(x) = \arctan\left(\dfrac{x^2}{\sqrt{3x+1}}\right)$ $\quad f'(x) = \dfrac{1}{\left(\left(\frac{x^2}{\sqrt{3x+1}}\right)^2+1\right)}\cdot\dfrac{\sqrt{3x+1}\cdot 2x - x^2\frac{1}{2\sqrt{3x+1}}\cdot 3}{(3x+1)}$

25. $f(x) = \dfrac{\sinh(x^2-2)}{x+\sin^{-1}x}$ $\quad f'(x) = \dfrac{(x+\sin^{-1}x)\cosh(x^2-2)(2x) - \sinh(x^2-2)\left(1+\frac{1}{\sqrt{1-x^2}}\right)}{(x+\sin^{-1}x)^2}$

26. $f(x) = \dfrac{5\sinh x\tanh x}{\cosh x}$ $\quad f'(x) = \dfrac{\cosh x\left[5\sinh x\,\mathrm{sech}^2 x + \tanh x\cdot 5\cosh x\right] - 5\sinh x\tanh x(\sinh x)}{\cosh^2 x}$

27. $f(x) = \dfrac{\sec(5x^2)}{\arctan\left(\frac{x}{3}\right)}$ $\quad f'(x) = \dfrac{\arctan(x/3)\sec(5x^2)\tan(5x^2)\cdot 10x - \sec(5x^2)\frac{1}{(x/3)^2+1}\cdot(1/3)}{[\arctan(x/3)]^2}$

28. $f(x) = \dfrac{\arctan(5x)}{\tanh(10x-1)}$ $\quad f'(x) = \dfrac{\tanh(10x-1)\cdot\frac{1}{(5x)^2+1}\cdot 5 - \arctan(5x)\,\mathrm{sech}^2(10x-1)(10)}{[\tanh(10x-1)]^2}$

29. $f(x) = \sec^{-1}(3x)$ $\quad f'(x) = \dfrac{1}{3x\sqrt{(3x)^2-1}}\cdot 3$

30. $f(x) = \tanh^{-1}\left(\dfrac{1}{\cos x}\right)$ $\quad f'(x) = \dfrac{1}{1-\left(\frac{1}{\cos x}\right)^2}\cdot\sec x\tan x$

rewrite $\rightarrow$ sec

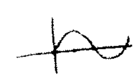
31. $f(x) = \cosh\left(e^{\arccos e^x}\right)$ $\quad f'(x) = \sinh\left(e^{\arccos e^x}\right)e^{\arccos e^x}\left(\dfrac{-1}{\sqrt{1-(e^x)^2}}\right)\cdot e^x$

Limits: Compute each of the following limit.

32. $\displaystyle\lim_{x\to1}\frac{5x-5}{\ln x\cdot\cos x}$ $\quad \frac{0}{0} \overset{L'H}{=} \lim_{x\to1}\frac{5}{\ln x(-\sin x)+\cos x\cdot\frac{1}{x}} = \frac{5}{0+\cos 1} = \boxed{\frac{5}{\cos 1}}$

33. $\displaystyle\lim_{x\to0}\frac{\sin(3x)}{9\cos x-5x-9}$ $= \frac{0}{0} \overset{L'H}{=} \lim_{x\to0}\frac{3\cos(3x)}{-9\sin x-5} = \boxed{-\frac{3}{5}}$

34. $\displaystyle\lim_{x\to1}\frac{\cos\left(\frac{\pi}{2}x\right)}{x^2-x}$ $\quad \frac{0}{0} \overset{L'H}{=} \lim_{x\to1}\frac{-\sin\left(\frac{\pi x}{2}\right)\cdot\frac{\pi}{2}}{2x-1} = \frac{-\pi/2}{1} = \boxed{-\frac{\pi}{2}}$



35. $\displaystyle\lim_{x\to3}\frac{\sin(x-3)}{x^2-9}$ $\quad \frac{0}{0} \overset{L'H}{=} \lim_{x\to3}\frac{\cos(x-3)}{2x} = \boxed{\frac{1}{6}}$

36. $\displaystyle\lim_{x\to\infty}\frac{5x^2+7x}{3x^2+x}$ $\quad \frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\infty}\frac{10x+7}{6x+1} \overset{L'H}{=} \lim_{x\to\infty}\frac{10}{6} = \boxed{\frac{5}{3}}$

37. $\displaystyle\lim_{x\to\infty}\frac{x^2-3x}{e^x-e^{-x}}$ $\quad \frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\infty}\frac{2x-3}{e^x+e^{-x}} = \lim_{x\to\infty}\frac{2}{e^x+1/e^x} = \boxed{0}$

38. $\displaystyle\lim_{x\to0}(1-\sin(2x))^{\frac{1}{x}}$

$= e^{\lim_{x\to0}\ln(1-\sin(2x))^{1/x}} = e^{\lim_{x\to0}\frac{1}{x}\ln(1-\sin(2x))} \overset{L'H}{=} e^{\lim_{x\to0}\frac{\frac{1}{1-\sin(2x)}\cdot -2\cos(2x)}{1}} = \boxed{e^{-2}}$

• 39. $\lim_{x\to 3} \frac{x^2-9}{3-x}\ \frac{0}{0} = \lim_{x\to 3} \frac{(x-3)(x+3)}{-(x-3)} = \boxed{-6}$ or $\overset{L'H}{=} \lim_{x\to 3} \frac{2x}{-1} = \boxed{-6}$

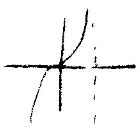
• 40. $\lim_{x\to 0^+} \left(\frac{1}{e^x-1} - \frac{1}{x}\right)\ \infty-\infty = \lim_{x\to 0^+} \frac{x-(e^x-1)}{x(e^x-1)} = \lim_{x\to 0^+} \frac{x-e^x+1}{x(e^x-1)}\ \frac{0}{0} \overset{L'H}{=} \lim_{x\to 0^+} \frac{1-e^x}{xe^x+(e^x-1)} \overset{L'H}{=} \lim_{x\to 0^+} \frac{-e^x}{xe^x+e^x+e^x}$

$= \lim_{x\to 0^+} \frac{-\cancel{e^x}}{\cancel{e^x}(x+2)} = \boxed{-\tfrac{1}{2}}$

• 41. $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e^{\lim_{x\to\infty} \ln(1+1/x)^x} = e^{\lim_{x\to\infty} x\ln(1+1/x)} = e^{\lim_{x\to\infty} \frac{\ln(1+1/x)}{1/x}}\ \frac{0}{0}$

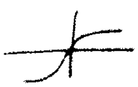
$\overset{L'H}{=} e^{\lim_{x\to\infty} \frac{\left(\frac{1}{1+1/x}\right)\cdot\cancel{\left(-1/x^2\right)}}{\cancel{-1/x^2}}} = e^1 = \boxed{e}$

• 42. $\lim_{x\to 0} \frac{e^x-1}{\ln(x+1)} \overset{L'H}{=} \lim_{x\to 0} \frac{e^x}{\left(\frac{1}{x+1}\right)(1)} = e^0 = \boxed{1}$



• 43. $\lim_{x\to\frac{\pi}{2}} \frac{\tan x}{1+\tan x}\ \frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\pi/2} \frac{\cancel{\sec^2 x}}{\cancel{\sec^2 x}} = \boxed{1}$

• 44. $\lim_{x\to 1} \frac{\ln x}{x^3-1}\ \frac{0}{0} \overset{L'H}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{3x^2} = \lim_{x\to 1} \frac{1}{3x^3} = \boxed{\frac{1}{3}}$



• 45. $\lim_{x\to\infty} \frac{\arctan x}{x} = \frac{\pi/2}{\infty} = \boxed{0}$

• 46. $\lim_{x\to 0^+} x^3 \ln x\ \ 0(-\infty) = \lim_{x\to 0^+} \frac{\ln x}{1/x^3} \overset{L'H}{=} \lim_{x\to 0^+} \frac{1/x}{-3/x^4} \cdot \frac{x^4}{x^4} = \lim_{x\to 0^+} \frac{x^3}{-3} = \boxed{0}$

• 47. $\lim_{x\to 0^+} x\ln x\ \ 0(-\infty) = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} -x = \boxed{0}$

• 48. $\lim_{x\to 0} \frac{e^{2x}-e^x}{\sin(3x)}\ \frac{0}{0} \overset{L'H}{=} \lim_{x\to 0} \frac{2e^{2x}-e^x}{3\cos(3x)} = \boxed{\frac{1}{3}}$

• 49. $\lim_{x\to 0} \frac{\sinh x}{3x}\ \frac{0}{0} \overset{L'H}{=} \lim_{x\to 0} \frac{\cosh x}{3} = \boxed{1/3}$

• 50. $\lim_{x\to 2} \frac{x-2+\sin(x-2)}{x^2-6x+8}\ \frac{0}{0} \overset{L'H}{=} \lim_{x\to 2} \frac{1+\cos(x-2)}{2x-6} = \frac{1+1}{-2} = \boxed{-1}$

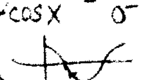
• 51. $\lim_{x\to 0} \frac{x\sin x}{\cos x-1}\ \frac{0}{0} \overset{L'H}{=} \lim_{x\to 0} \frac{x\cos x+\sin x}{-\sin x}\ \frac{0}{0} = \lim_{x\to 0} \frac{x(-\sin x)+\cos x+\cos x}{-\cos x} = \boxed{-2}$

• 52. $\lim_{x\to 0^+} x^{\sin x} = e^{\lim_{x\to 0} \ln(x^{\sin x})} = e^{\lim_{x\to 0} \sin x \ln x} = e^{\lim_{x\to 0} \frac{\ln x}{1/\sin x}} = e^{\lim_{x\to 0} \frac{1/x}{-\frac{1}{\sin^2 x}\cdot\cos x}} = e^{\lim_{x\to 0} \frac{-\sin^2 x}{x\cos x}}\ {}^{0/0} = e^{\lim_{x\to 0} \frac{-2\sin x\cos x}{x(-\sin x)+\cos x}}\ {}^{\to 0}$

$= e^0 = \boxed{1}$

• 53. $\lim_{x\to 0^+} (\cos x)^{\frac{1}{x}} = e^{\lim_{x\to 0^+} \frac{\ln\cos x}{x}}\ \frac{0}{0} \overset{L'H}{=} e^{\lim_{x\to 0^+} \frac{\frac{1}{\cos x}\cdot -\sin x}{1}} = e^{\lim_{x\to 0^+} -\tan x} = e^0 = \boxed{1}$

• 54. $\lim_{x\to\frac{\pi}{2}^+} \frac{\cos x}{1-\sin x}\ \frac{0}{0} = \lim_{x\to\pi/2^+} \frac{\cancel{-}\sin x}{\cancel{-}\cos x} = \frac{1}{0^-} = -\infty$



• 55. $\lim_{x\to 0^+} x\ln\left(\frac{1}{x}\right)\ \ 0\cdot\infty = \lim_{x\to 0^+} \frac{\ln(1/x)}{1/x} \overset{L'H}{=} \lim_{x\to 0^+} \frac{\cancel{(1/x)}\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-1/x^2}} = \lim_{x\to 0^+} x = \boxed{0}$

• 56. $\lim_{x\to 0^+} xe^{\frac{1}{x}}\ \ 0\cdot\infty = \lim_{x\to 0^+} \frac{e^{1/x}}{1/x} = \lim_{x\to 0^+} \frac{e^{1/x}\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}} = \lim_{x\to 0^+} e^{1/x} = e^{+\infty} = \boxed{+\infty}$

• 57. $\lim_{x\to 0^-} xe^{\frac{1}{x}}$ Same as 56 $\lim_{x\to 0^-} e^{1/x} = e^{-\infty} = \boxed{0}$

• 58. $\lim_{x\to 1} \frac{3\cos(1-x)-3x}{\sin(1-x)}\ \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 1} \frac{-3\sin(1-x)(-1)-3}{\cos(1-x)(-1)} = \lim_{x\to 1} \frac{3\sin(1-x)^{\to 0}-3}{-\cos(1-x)} = \frac{-3}{-1} = \boxed{3}$

59. $\lim_{x\to\infty} \frac{e^x}{\ln x}$ $\frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\infty} \frac{e^x}{1/x} = \lim_{x\to\infty} xe^x = \boxed{+\infty}$

60. $\lim_{x\to\infty} x^{\frac{1}{x^2}}$ ∞^0 $= e^{\lim_{x\to\infty} \frac{1}{x^2}\ln x}$ $\frac{\infty}{\infty} \overset{L'H}{=} e^{\lim_{x\to\infty} \frac{1/x}{2x}} = e^{\lim_{x\to\infty} \frac{1}{2x^2}} = e^0 = \boxed{1}$

61. $\lim_{x\to0^+} \frac{\ln x - 1}{\arcsin x}$ $\frac{-\infty}{0^+} = \boxed{-\infty}$ direct substitution

62. $\lim_{x\to 0}\frac{x}{\tan x}$ $\frac00 \overset{L'H}{=} \lim_{x\to0} \frac{1}{\sec^2 x} = \lim_{x\to0} \cos^2 x = \boxed{1}$

63. $\lim_{x\to1^+} x^{\frac{1}{x-1}}$ 1^∞ $= e^{\lim_{x\to1^+} \frac{\ln x}{x-1}}$ $\frac00 \overset{L'H}{=} e^{\lim_{x\to1^+} \frac{1/x}{1}} = e^1 = \boxed{e}$

64. $\lim_{x\to\pi} \frac{\cos x \sin x}{x-\pi}$ $\frac00 \overset{L'H}{=} \lim_{x\to\pi} \frac{\cos x\cdot\cos x + \sin x(-\sin x)}{1} = \frac{(-1)^2+0}{1} = \boxed{1}$

65. $\lim_{x\to\frac{\pi}{2}^-} \tan x - \sec x$ $\infty - \infty$ $= \lim_{x\to\frac{\pi}{2}^-} \frac{\sin x}{\cos x} - \frac{1}{\cos x} = \lim_{x\to\frac{\pi}{2}^-} \frac{\sin x - 1}{\cos x}$ $\frac00 \overset{L'H}{=} \lim_{x\to\frac{\pi}{2}^-} \frac{\cos x}{-\sin x} = \frac{0}{-1} = \boxed{0}$

66. $\lim_{x\to0^+}(1-2x)^{\frac1x}$ 1^∞ $= e^{\lim_{x\to0^+}\frac{\ln(1-2x)}{x}}$ $\frac00 \overset{L'H}{=} e^{\lim_{x\to0^+} \frac{\left(\frac{1}{1-2x}\right)\cdot(-2)}{1}} = \boxed{e^{-2}}$

67. $\lim_{x\to\infty}(x^2+1)^{\frac{1}{\ln x}}$ ∞^0 $= e^{\lim_{x\to\infty} \frac{\ln(x^2+1)}{\ln x}}$ $\frac\infty\infty \overset{L'H}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{x^2+1}\cdot 2x}{1/x}} = e^{\lim_{x\to\infty} \frac{2x^2}{x^2+1}}$ $\frac\infty\infty = \boxed{e^2}$

68. $\lim_{x\to\infty}(e^x+1)^{\frac1x}$ ∞^0 $= e^{\lim_{x\to\infty} \frac{\ln(e^x+1)}{x}}$ $\frac\infty\infty \overset{L'H}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{e^x+1}\cdot e^x}{1}}$ $\frac\infty\infty \overset{L'H}{=} e^{\lim_{x\to\infty} \frac{e^x}{e^x}} = e^1 = \boxed{e}$

69. $\lim_{x\to\infty}\left(\cos\frac1x\right)^x$ 1^∞ $= e^{\lim_{x\to\infty} x \ln\cos\frac1x}$ $\infty\cdot0$ $= e^{\lim_{x\to\infty}\frac{\ln(\cos\frac1x)}{1/x}}$ $\frac00 \overset{L'H}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{\cos 1/x}(-\sin\frac1x)(-\frac{1}{x^2})}{-1/x^2}} = e^{\frac01} = e^0 = \boxed{1}$

70. $\lim_{x\to\infty}(x^3+1)^{\frac{1}{\ln x}}$ ∞^0 $= e^{\lim_{x\to\infty} \frac{\ln(x^3+1)}{\ln x}}$ $\frac\infty\infty \overset{L'H}{=} e^{\lim_{x\to\infty}\frac{\frac{1}{x^3+1}\cdot 3x^2}{1/x}} = e^{\lim_{x\to\infty}\frac{3x^3}{x^3+1}} \overset{L'H}{=} e^{\lim_{x\to\infty} \frac{9x^2}{3x^2}} = \boxed{e^3}$

71. $\lim_{x\to0^+}\frac{1}{\ln(x+1)} - \frac1x$ $\infty-\infty$ $= \lim_{x\to0^+} \frac{x-\ln(x+1)}{x\ln(x+1)}$ $\frac00 \overset{L'H}{=} \lim_{x\to0^+} \frac{1-\frac{1}{x+1}}{x\frac{1}{x+1}+\ln(x+1)}\cdot\frac{(x+1)}{(x+1)} = \lim_{x\to0^+} \frac{(x+1)-1}{x+(x+1)\ln(x+1)} = \lim_{x\to0^+} \frac{x}{x+x+\ldots}$ $= \lim_{x\to0^+}\frac{1}{1+(x+1)\left(\frac{1}{x+1}\right)+\ln(x+1)} = \boxed{\frac12}$

72. $\lim_{x\to1^+}\frac{1}{\ln x}-\frac{x}{x-1}$ $\infty-\infty$ $= \lim_{x\to1^+} \frac{x-1-x\ln x}{(x-1)\ln x}$ $\frac00 \overset{L'H}{=}\lim_{x\to1^+} \frac{1-x\frac1x-\ln x}{(x-1)\frac1x+\ln x}$ $\frac00 = \lim_{x\to1^+} \frac{-1/x}{\frac{1}{x^2}+\frac1x}$ $= \lim_{x\to1^+}\frac{-x}{1+x} = \boxed{-\frac12}$

$1-\frac1x+\ln x$

73. $\lim_{x\to0^+}(1+\sinh x)^{\frac{1}{\sqrt{x}}}$ 1^∞ $= e^{\lim_{x\to0} \frac{\ln(1+\sinh x)}{\sqrt x}}$ $\frac00 = e^{\lim_{x\to0} \frac{\frac{1}{1+\sinh x}\cosh x}{\frac{1}{2\sqrt x}}}$ $= e^{\lim_{x\to0}\frac{2\sqrt x}{(1+\sinh x)\cosh x}} = e^0 = \boxed{1}$

13.

74. $\lim_{x\to\infty} x^2\sin\left(\frac{1}{x^2}\right)$ $\infty\cdot0$ $= \lim_{x\to\infty}\frac{\sin(1/x^2)}{1/x^2}$ $\frac00 = \lim_{x\to\infty} \frac{\cos(1/x^2)(-\frac{2}{x^3})}{-\frac{2}{x^3}} = \cos 0 = \boxed{1}$

75. $\lim_{x\to0^+}\sqrt x\ln x$ $0\cdot(-\infty)$ $= \lim_{x\to0^+}\frac{\ln x}{\left(\frac{1}{\sqrt x}\right)} = \lim_{x\to0^+}\frac{1/x}{-\frac{1}{2x^{3/2}}} = \lim_{x\to0^+} -\frac{x^{3/2}}{x} = \lim_{x\to0} -\sqrt x = \boxed{0}$

76. $\lim_{x\to1}\frac{e^{x^2}-e^x}{\ln x}$ $\frac00 \overset{L'H}{=} \lim_{x\to1}\frac{2xe^{x^2}-e^x}{1/x} = \frac{2e^1-e^1}{1} = \boxed{e^1}$

77. $\lim_{x\to0}(1+3x)^{\frac{2}{x}}$ 1^∞ $= e^{\lim_{x\to0}\frac{2\ln(1+3x)}{x}}$ $\frac00 \overset{L'H}{=} e^{\lim_{x\to0} \frac{2\cdot\frac{1}{1+3x}(3)}{1}} = \boxed{e^6}$

78. $\lim_{x\to\infty}x(2e^{\frac1x}-2)$ $\infty\cdot0$ $= \lim_{x\to\infty}\frac{2e^{1/x}-2}{1/x}$ $\frac00 \overset{L'H}{=}\lim_{x\to\infty}\frac{2e^{1/x}(-\frac{1}{x^2})}{-\frac{1}{x^2}} = 2e^0 = \boxed{2}$

79. $\lim_{x\to\infty}\left(1-\frac3x\right)^{4x}$

$= e^{\lim_{x\to\infty} 4x\ln(1-\frac3x)}$ $\infty\cdot0$ $= e^{\lim_{x\to\infty}\frac{\ln(1-3/x)}{1/4x}}$ $\frac00 \overset{L'H}{=} e^{\lim_{x\to\infty}\frac{\frac{1}{(1-3/x)}\left(+\frac{3}{x^2}\right)}{-\frac{1}{4x^2}}} = e^{\lim_{x\to\infty}\frac{-12}{1-3/x}} = \boxed{e^{-12}}$

• 80. $\lim_{x\to 0^+} (\cos x)^{\frac{1}{x^2}}$ 1^∞ $= e^{\lim_{x\to 0^+} \ln(\cos x)^{1/x^2}} = e^{\lim_{x\to 0^+} \frac{1}{x^2}\ln\cos x} = e^{\lim_{x\to 0^+} \frac{\ln\cos x}{x^2}}$ $\frac{0}{0}$ $\overset{L'H}{=} e^{\lim_{x\to 0^+} \frac{\frac{1}{\cos x}(-\sin x)}{2x}} = e^{\lim_{x\to 0^+} \frac{-\sin x}{2x\cos x}}$

• 81. $\lim_{x\to 0^+} (\cos \sqrt{x})^{\frac{1}{x}} = e^{\lim_{x\to 0^+} \ln(\cos\sqrt{x})^{1/x}} = e^{\lim_{x\to 0^+} \frac{1}{x}\ln\cos\sqrt{x}} = e^{\lim_{x\to 0^+} \frac{\ln\cos\sqrt{x}}{x}}$ $\frac{0}{0}$ $\overset{L'H}{=} e^{\lim_{x\to 0^+} \frac{\frac{1}{\cos\sqrt{x}}(-\sin\sqrt{x})\frac{1}{2\sqrt{x}}}{1}}$ $\overset{L'H}{=} e^{\lim_{x\to 0^+} \frac{-\cos x}{2x - \sin x}}$

• 82. $\lim_{x\to\infty} (e^x + x)^{\frac{1}{x}}$ ∞^0 $= e^{\lim_{x\to\infty} \frac{\ln(e^x+x)}{x}}$ $\frac{\infty}{\infty}$ $\overset{L'H}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{e^x+x}(e^x+1)}{1}}$ $\overset{L'H}{=} e^{\lim_{x\to\infty} \frac{e^x}{e^x+1}}$ $\frac{\infty}{\infty}$ $\overset{L'H}{=} e^{\lim_{x\to\infty} \frac{\cancel{e^x}}{\cancel{e^x}}} = e^1 = \boxed{e}$

$= e^{\lim_{x\to 0^+} \frac{-\sin\sqrt{x}}{2\sqrt{x}\cos\sqrt{x}}}$ $\frac{0}{0}$

$= e^{\lim_{x\to 0^+} \frac{-\cos\sqrt{x}\,\frac{1}{2\sqrt{x}}}{2\sqrt{x}(-\sin\sqrt{x})\frac{1}{2\sqrt{x}} + \cos\sqrt{x}\cdot\frac{2}{2\sqrt{x}}}\cdot\left(\frac{\sqrt{x}}{\sqrt{x}}\right)}$

$= e^{\lim_{x\to 0^+} \frac{-\cos\sqrt{x}\cdot\frac{1}{2}}{\sqrt{x}(-\sin\sqrt{x})+\cos\sqrt{x}}}$

$= \boxed{e^{-1/2}}$ ✓

$= e^{\frac{-1}{2}} = \boxed{e^{-1/2}}$

• 83. $\lim_{x\to 0} \frac{\sinh^{-1} x}{x}$ $\frac{0}{0}$ $\overset{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{\sqrt{1+x^2}}}{1} = \boxed{1}$ ✓

Integrals: Compute each of the following integrals.

84. $\int (e^x + x)^2 \, dx$

85. $\int \frac{\sec^2(3x)}{\sqrt{1 + \tan^2(3x)}} \, dx$

86. $\int (x + 7)e^{2x+3} \, dx$

87. $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} x \sec^2 x \, dx$

88. $\int \frac{1 + x}{\sqrt{x^2 - 1}} \, dx$

89. $\int x \sin^2 x \, dx$

90. $\int \tan^2 x \cos^4 x \, dx$

91. $\int \frac{1}{\sqrt{1 - 25x^2}} \, dx$

92. $\int \frac{1}{\sqrt{25 - x^2}} \, dx$

93. $\int \frac{1}{x^2 + 25} \, dx$

94. $\int \frac{1}{25x^2 + 1} \, dx$

95. $\int_0^{\ln 4} x^2 \cosh x \, dx$

96. $\int \frac{1}{x\sqrt{9 - \ln^2 x}} \, dx$

97. $\int_0^{\frac{\pi}{4}} x \cos x - x \sin x \, dx$

$\frac{1}{\sqrt{e}} = e$

or

Simplify

$= e^{\lim_{x\to 0^+} \frac{-\tan\sqrt{x}}{2\sqrt{x}}}$ $\frac{0}{0}$

$\overset{L'H}{=} e^{\lim_{x\to 0^+} \frac{-\sec^2\sqrt{x}\,\frac{1}{2\sqrt{x}}}{2\cdot\frac{1}{2\sqrt{x}}}}$

$= e^{\lim_{x\to 0^+} -\frac{1}{2}\sec^2\sqrt{x}}$

$= \boxed{e^{-1/2}}$ ✓ #81

Integrals

84. $\int (e^x+x)^2 dx = \int e^{2x} + 2xe^x + x^2\, dx = \frac{1}{2}e^{2x} + 2\int xe^x dx + \frac{x^3}{3} + C$

$\boxed{\begin{array}{ll} u=x & dv=e^x dx \\ du=dx & v=e^x \end{array}}$

$= \frac{1}{2}e^{2x} + 2\left[xe^x - \int e^x dx\right] + \frac{x^3}{3} + C$

$= \boxed{\frac{1}{2}e^{2x} + 2xe^x - 2e^x + x^3/3 + C}$ ✓

85. $\int \frac{\sec^2(3x)}{\sqrt{1+\tan^2(3x)}} dx = \frac{1}{3}\int \frac{1}{\sqrt{1+u^2}} du = \frac{1}{3}\sinh^{-1}u + C = \boxed{\frac{1}{3}\sinh^{-1}(\tan(3x)) + C}$

$\boxed{\begin{array}{l} u=\tan(3x) \\ du=3\sec^2(3x)\,dx \\ \frac{1}{3}du = \sec^2(3x)\,dx \end{array}}$

or notice $\int \frac{\sec^2(3x)}{\sqrt{\sec^2(3x)}} dx = \int \sec(3x)\,dx = \boxed{\frac{1}{3}\ln|\sec(3x)+\tan(3x)| + C}$

86. $\int (x+7)e^{2x+3} dx = (x+7)\frac{1}{2}e^{2x+3} - \frac{1}{2}\int e^{2x+3} dx = \boxed{\frac{(x+7)e^{2x+3}}{2} - \frac{1}{4}e^{2x+3} + C}$ ✓

$\boxed{\begin{array}{ll} u=x+7 & dv=e^{2x+3}dx \\ du=dx & v=\frac{1}{2}e^{2x+3} \end{array}}$

87. $\int_{\pi/6}^{\pi/4} x\sec^2 x\,dx = x\tan x \Big|_{\pi/6}^{\pi/4} - \int_{\pi/6}^{\pi/4} \tan x\,dx = x\tan x + \ln|\cos x| \Big|_{\pi/6}^{\pi/4}$

$u=x \quad dv=\sec^2 x\,dx$
$du=dx \quad v=\tan x$

$= \left(\frac{\pi}{4}\tan(\frac{\pi}{4}) + \ln|\cos\frac{\pi}{4}|\right) - \left(\frac{\pi}{6}\tan(\frac{\pi}{6}) + \ln|\cos\frac{\pi}{6}|\right)$

$= \frac{\pi}{4} + \ln\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi}{6}\,\frac{\sin\pi/6}{\cos\pi/6} - \ln\frac{\sqrt{3}}{2}$

$-\ln\sqrt{2} \qquad \frac{1}{\sqrt{3}}$

$= \frac{\pi}{4} - \frac{\ln 2}{2} - \frac{\pi}{6\sqrt{3}} - (\ln\sqrt{3} - \ln 2)$

$= \frac{\pi}{4} - \frac{\ln 2}{2} - \frac{\pi}{6\sqrt{3}} - \frac{1}{2}\ln 3 + \ln 2$

$= \boxed{\frac{\pi}{4} + \frac{\ln 2}{2} - \frac{\pi}{6\sqrt{3}} - \frac{1}{2}\ln 3}$ ✓

2. $\int \frac{1+x}{\sqrt{x^2-1}} dx = \int \frac{1}{\sqrt{x^2-1}} + \frac{x}{\sqrt{x^2-1}} dx$

$u=x^2-1$
$du=2x\,dx$
$\frac{1}{2}du = x\,dx$
$\frac{1}{2}\int \frac{1}{\sqrt{u}} du$
$\frac{1}{2}2u^{1/2}$

$= \boxed{\cosh^{-1}x + \sqrt{x^2-1} + C}$ ✓

1. $\int x\sin^2 x\,dx = x\frac{1}{2}\left[x - \frac{1}{2}\sin(2x)\right] - \frac{1}{2}\int x - \frac{1}{2}\sin(2x)\,dx$

$\boxed{\begin{array}{ll} u=x & dv=\sin^2 x\,dx \\ du=dx & v=\int \sin^2 x\,dx \\ & =\frac{1}{2}\int 1-\cos(2x)\,dx \\ & =\frac{1}{2}\left[x-\frac{1}{2}\sin(2x)\right] \end{array}}$

$= \frac{x^2}{2} - \frac{x\sin(2x)}{4} - \frac{1}{2}\left[\frac{x^2}{2} + \frac{1}{4}\cos(2x)\right] + C$

$= \frac{x^2}{2} - \frac{x\sin(2x)}{4} - \frac{x^2}{4} - \frac{1}{8}\cos(2x) + C$

$= \boxed{\frac{x^2}{4} - \frac{x\sin(2x)}{4} - \frac{1}{8}\cos(2x) + C}$ ✓

90. $\int \tan^2 x \cos^4 x \, dx = \int \frac{\sin^2 x}{\cos^2 x} \cos^4 x \, dx = \int \sin^2 x \cos^2 x \, dx = \int \left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1+\cos(2x)}{2}\right) dx$

$= \frac{1}{4} \int 1 - \cos^2(2x) \, dx = \frac{1}{4}\int 1 - \left[\frac{1+\cos(4x)}{2}\right] dx$

$= \frac{1}{4}\int \frac{1}{2} - \frac{1}{2}\cos(4x) \, dx$

$= \frac{1}{8}\int 1 - \cos(4x) \, dx$

$= \boxed{\frac{1}{8}\left(x - \frac{1}{4}\sin(4x)\right) + C}$ ✓

91. $\int \frac{1}{\sqrt{1-25x^2}} dx = \int \frac{1}{\sqrt{1-(5x)^2}} dx$

$\boxed{\begin{array}{l} u = 5x \\ du = 5dx \\ \frac{1}{5}du = dx \end{array}}$

$= \frac{1}{5}\int \frac{1}{\sqrt{1-u^2}} du = \frac{1}{5}\arcsin u + C = \boxed{\frac{1}{5}\arcsin(5x) + C}$ ✓

92. $\int \frac{1}{\sqrt{25-x^2}} dx = \int \frac{1}{\sqrt{25(1-(\frac{x}{5})^2)}} dx = \frac{1}{5}\int \frac{1}{\sqrt{1-(\frac{x}{5})^2}} dx = \int \frac{1}{\sqrt{1-w^2}} dw = \arcsin w + C = \boxed{\arcsin\left(\frac{x}{5}\right) + C}$ ✓

$w = \frac{x}{5}$

$dw = \frac{1}{5}dx$

93. $\int \frac{1}{x^2+25} dx = \int \frac{1}{25((\frac{x}{5})^2+1)} dx = \frac{5}{25}\int \frac{1}{w^2+1} dw = \boxed{\frac{1}{5}\arctan\left(\frac{x}{5}\right) + C}$ ✓

$w = \frac{x}{5}$

$dw = \frac{1}{5}dx$

$5dw = dx$

94. $\int \frac{1}{25x^2+1} dx = \int \frac{1}{(5x)^2+1} dx = \frac{1}{5}\int \frac{1}{w^2+1} dw = \boxed{\frac{1}{5}\arctan(5x) + C}$ ✓

$w = 5x$

$dw = 5dx$

$\frac{1}{5}dw = dx$

95. $\int_0^{\ln 4} x^2 \cosh x \, dx = x^2 \sinh x \Big|_0^{\ln 4} - \int_0^{\ln 4} 2x \sinh x \, dx = x^2 \sinh x \Big|_0^{\ln 4} - 2\left[x\cosh x \Big|_0^{\ln 4} - \int_0^{\ln 4} \cosh x \, dx\right]$

$\boxed{\begin{array}{ll} u = x^2 & dv = \cosh x \, dx \\ du = 2x\,dx & v = \sinh x \end{array}}$ $\boxed{\begin{array}{ll} u = x & dv = \sinh x \, dx \\ du = dx & v = \cosh x \end{array}}$

$x^2 \sinh x - 2x\cosh x + 2\sinh x \Big|_0^{\ln 4}$

$\frac{e^{\ln 4} - \frac{1}{e^{\ln 4}}}{2} = \frac{4 - \frac{1}{4}}{2} = \frac{15/4}{2} = \frac{15}{8}$

$= (\ln 4)^2 \sinh(\ln 4) - 2\ln 4 \cosh \ln 4 + 2\sinh \ln 4 - [\overset{0}{0 \sinh 0} - \overset{0}{2 \cdot 0 \cosh 0} + \overset{0}{2\sinh 0}]$

$\boxed{15(\ln 4)^2 - 34\ln 4 + 30}$ ✓

96 $\displaystyle\int \frac{1}{x\sqrt{9-\ln^2 x}}\,dx = \int \frac{1}{\sqrt{9-u^2}}\,du = \int \frac{1}{\sqrt{9(1-(\frac{u}{3})^2)}}\,du = \frac{1}{3}\int \frac{1}{\sqrt{1-(\frac{u}{3})^2}}\,du = \int \frac{1}{\sqrt{1-w^2}}\,dw$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$w = \frac{u}{3}$

$dw = \frac{1}{3}\,du$

$= \arcsin w + C$

$= \arcsin \frac{u}{3} + C$

$= \boxed{\arcsin\left(\frac{\ln x}{3}\right) + C}$

✓

97. $\displaystyle\int_0^{\pi/4} x\cos x - x\sin x\,dx = \int_0^{\pi/4} x\cos x\,dx - \int_0^{\pi/4} x\sin x\,dx$

$\boxed{\begin{array}{ll} u = x & dv = \cos x\,dx \\ du = dx & v = \sin x \end{array}}$ $\boxed{\begin{array}{ll} u = x & dv = \sin x\,dx \\ du = dx & v = -\cos x \end{array}}$

$\displaystyle = x\sin x\Big|_0^{\pi/4} - \int_0^{\pi/4}\sin x\,dx - \left[x(-\cos x)\Big|_0^{\pi/4} - \int_0^{\pi/4} -\cos x\,dx\right]$

$= x\sin x + \cos x + x\cos x - \sin x\Big|_0^{\pi/4}$

$= \frac{\pi}{4}\sin\frac{\pi}{4} + \cos\frac{\pi}{4} + \frac{\pi}{4}\cos\frac{\pi}{4} - \sin\frac{\pi}{4} - [\cancel{0\sin 0}^{\,0} + \cos 0 + \cancel{0\cos 0}^{\,0} - \cancel{\sin 0}^{\,0}]$

$= \frac{\pi}{4}\frac{\sqrt{2}}{2} + \cancel{\frac{\sqrt{2}}{2}} + \frac{\pi}{4}\frac{\sqrt{2}}{2} - \cancel{\frac{\sqrt{2}}{2}} - 1$

$= \boxed{\frac{\sqrt{2}\pi}{4} - 1}$ ✓

98 $\displaystyle\int \frac{e^{3x}}{1+e^{2x}}\,dx = \int \frac{e^{2x}e^x}{1+(e^x)^2}\,dx = \int \frac{u^2}{1+u^2}\,du = \int \frac{u^2+1-1}{1+u^2}\,du = \int \frac{u^2+1}{u^2+1} - \frac{1}{1+u^2}\,du$

$u = e^x$

$du = e^x\,dx$

✓

$\displaystyle = \int 1 - \frac{1}{1+u^2}\,du = u - \arctan u + C$

$= \boxed{e^x - \arctan e^x + C}$

check!

99. $\displaystyle\int x\sin^3 x\cos^2 x\,dx = x\left[-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5}\right] - \int -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5}\,dx$

$u = x \quad dv = \sin^3 x\cos^2 x\,dx$

$du = dx \quad v = \displaystyle\int \sin^3 x\cos^2 x\,dx$

$\displaystyle = \int \sin^2 x\cos^2 x\sin x\,dx$

$\displaystyle \int (1-\cos^2 x)\cos^2 x\sin x\,dx$

$w = \cos x$

$dw = -\sin x\,dx$

$-dw = \sin x\,dx$

$\displaystyle -\int (1-w^2)w^2\,dw$

$\ldots - \frac{w^3}{3} + \frac{w^5}{5} = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5}$

carry down

$\displaystyle +\frac{1}{3}\int \cos^3 x\,dx - \frac{1}{5}\int \cos^5 x\,dx$

$\cos^2 x\cos x \qquad \cos^4\cos$

$\displaystyle +\frac{1}{3}\int (1-\sin^2 x)\cos x\,dx - \frac{1}{5}\int (1-\sin^2 x)^2\cos x\,dx$

$w = \sin x$

$dw = \cos x$

$\displaystyle +\frac{1}{3}\int 1-w^2\,dw - \frac{1}{5}\int (1-w^2)^2\,dw$

$1 - 2w^2 + w^4$

$+\frac{1}{3}\left(w - \frac{w^3}{3}\right) - \frac{1}{5}\left(w - \frac{2w^3}{3} + \frac{w^5}{5}\right) + C$

$\ldots + \frac{1}{3}\left(\sin x - \frac{\sin^3 x}{3}\right) - \frac{1}{5}\left(\sin x - \frac{2}{3}\sin^3 x + \frac{\sin^5 x}{5}\right) + C$

Final Answer: $\boxed{= x\left[-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5}\right] + \frac{1}{3}\left(\sin x - \frac{\sin^3 x}{3}\right) - \frac{1}{5}\left(\sin x - \frac{2}{3}\sin^3 x + \frac{\sin^5 x}{5}\right) + C}$

100 $\int \arctan\left(\frac{1}{x}\right)dx$ $= x\arctan\left(\frac{1}{x}\right) - \int x \cdot \frac{1}{1+\left(\frac{1}{x}\right)^2}\left(-\frac{1}{x^2}\right)dx$

$$\boxed{\begin{array}{ll} u = \arctan\left(\frac{1}{x}\right) & dv = dx \\ du = \frac{1}{1+\left(\frac{1}{x}\right)^2}\left(-\frac{1}{x^2}\right) & v = x \end{array}}$$

$= x\arctan\left(\frac{1}{x}\right) + \int \frac{1}{x\left(1+\frac{1}{x^2}\right)}dx$

$u = \frac{1}{x^2}$

$du = -\frac{2}{x^3}dx$

$= x\arctan\left(\frac{1}{x}\right) + \int \frac{1}{u(u+1)}du$

$\frac{1}{x} \cdot x^2$

Partial Fractions

Try Again

$+ \int \frac{1}{x\left(\frac{x^2+1}{x^2}\right)}dx$ flip x

$+ \int \frac{x}{x^2+1}dx$

$u = x^2+1$ u-sub

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$+\frac{1}{2}\int \frac{1}{u}du$

$= x\arctan x + \frac{1}{2}\ln|u| + C$

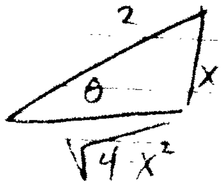
$= \boxed{x\arctan x + \frac{1}{2}\ln(x^2+1) + C}$

101. $\displaystyle\int \frac{1}{(4-x^2)^{3/2}}\,dx = \int \frac{1}{\left(\sqrt{4-4\sin^2\theta}\right)^3}\, 2\cancel{\cos}\theta\, d\theta = \frac{1}{4}\int \frac{1}{\cos^2\theta}\,d\theta = \frac{1}{4}\int \sec^2\theta\, d\theta$

$\left(\sqrt{4-4\sin^2\theta}\right)^3 = (2\cancel{\cos}\theta)^3$ $\quad \frac{2}{8}$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta \rightarrow \sin\theta = \frac{x}{2}$



$= \frac{1}{4}\tan\theta + C$

$= \boxed{\frac{1}{4}\left(\frac{x}{\sqrt{4-x^2}}\right) + C}$

✓

102. $\displaystyle\int x\arctan(3x)\,dx = \frac{x^2}{2}\arctan(3x) - \frac{3}{2}\int \frac{x^2}{9x^2+1}\,dx$

$\boxed{\begin{array}{ll} u = \arctan(3x) & dv = x\,dx \\ du = \frac{3}{9x^2+1}\,dx & v = \frac{x^2}{2} \end{array}}$

$-\frac{3}{2}\cdot\frac{1}{9}\int \frac{x^2}{x^2+\frac{1}{9}}\,dx$

$-\frac{1}{6}\int \frac{(x^2+\frac{1}{9})-\frac{1}{9}}{x^2+\frac{1}{9}}\,dx$

Tricky!

$-\frac{1}{6}\int 1 - \frac{1}{9(x^2+\frac{1}{9})}\,dx$

$-\frac{1}{6}\int 1 - \frac{1}{9x^2+1}\,dx$

$-\frac{1}{6}\int 1 - \frac{1}{(3x)^2+1}\,dx$

$w = 3x$

$dw = 3dx \Rightarrow \frac{1}{3}dw = dx$

$-\frac{1}{6}\int 1 - \frac{1}{3}\frac{1}{w^2+1}\,dw$

$= \boxed{\frac{x^2}{2}\arctan(3x) - \frac{1}{6}\left[x - \frac{1}{3}\arctan(3x)\right] + C}$

trig sub.

or $\to -\frac{3}{2}\int \frac{x^2}{(3x)^2+1}$ $\qquad$ let $3x = \tan\theta$, $x = \frac{\tan\theta}{3}$, $dx = \frac{1}{3}\sec^2\theta$

$-\frac{3}{2}\int \frac{\frac{\tan^2\theta}{9}}{\cancel{\sec^2\theta}}\,\frac{1}{3}\cancel{\sec^2\theta}\,d\theta$

$-\frac{1}{18}\int \tan^2\theta\, d\theta$

$-\frac{1}{18}\int \sec^2\theta + 1\, d\theta$

$= \frac{1}{18}\left[\tan\theta + \theta\right] + C$

$= -\frac{1}{18}\left[3x + \arctan(3x)\right] + C$

✓

103. $\displaystyle\int \arcsin x\, \frac{\ln\arcsin x}{\sqrt{1-x^2}}\,dx = \int w\ln w\, dw = \frac{w^2}{2}\ln w - \int \frac{w^2}{2}\,\frac{1}{w}\,dw = \frac{w^2\ln w}{2} - \frac{1}{2}\int w\, dw$

$W = \arcsin x$

$dw = \frac{1}{\sqrt{1-x^2}}\,dx$

$\boxed{\begin{array}{ll} u = \ln w & dv = w\,dw \\ du = \frac{1}{w}dw & v = \frac{w^2}{2} \end{array}}$

$= \frac{w^2\ln w}{2} - \frac{w^2}{4} + C$ ✓

$= \boxed{\frac{(\arcsin x)^2\ln(\arcsin x)}{2} - \frac{(\arcsin x)^2}{4} + C}$

104. $\int_1^e \ln x\,dx = x\ln x\Big|_1^e - \int_1^e \cancel{x}\,\frac{1}{\cancel{x}}\,dx = x\ln x\Big|_1^e - \int_1^e dx$

$$\boxed{\begin{array}{ll} u=\ln x & dv=dx \\ du=\frac{1}{x}dx & v=x \end{array}}$$

$= x\ln x - x \Big|_1^e$

$= (e\ln e - e) - (1\ln 1 - 1)$ ✓

$= e - e + 1$

$= \boxed{1}$

105. $\int \frac{\ln(2x^5)}{x^2}dx = -\frac{1}{x}\ln(2x^5) - \int\left(-\frac{1}{x}\right)\cdot\frac{5}{x}\,dx = -\frac{\ln(2x^5)}{x} + 5\int x^{-2}dx$

$$\boxed{\begin{array}{ll} u=\ln(2x^5) & dv=\frac{1}{x^2}dx \\ du=\frac{1}{2x^5}10x^4dx = \frac{5}{x}dx & v=\frac{x^{-1}}{-1}=-\frac{1}{x} \end{array}}$$

$= -\frac{\ln(2x^5)}{x} + \frac{5x^{-1}}{-1} + C$

$= \boxed{-\frac{\ln(2x^5)}{x} - \frac{5}{x} + C}$ ✓

106. $\int \ln^2(x^{20})\,dx$

$$\boxed{\begin{array}{ll} u=\ln^2(x^{20}) & dv=dx \\ du=2\ln(x^{20})\frac{1}{x^{20}}\cdot 20x^{19}dx & v=x \end{array}}$$

$= \frac{40\ln(x^{20})}{x}dx$

$= x\ln^2(x^{20}) - \int \cancel{x}\cdot\frac{40\ln(x^{20})}{\cancel{x}}dx$

$= x\ln^2(x^{20}) - 40\int \ln(x^{20})\,dx$

$$\boxed{\begin{array}{ll} u=\ln(x^{20}) & dv=dx \\ du=\frac{1}{x^{20}}20x^{19}dx = \frac{20}{x}dx & v=x \end{array}}$$

$= x\ln^2(x^{20}) - 40\left[x\ln(x^{20}) - \int \cancel{x}\left(\frac{20}{\cancel{x}}\right)dx\right]$

$= \boxed{x\ln^2(x^{20}) - 40x\ln(x^{20}) + 800x + C}$ ✓

107. $\int \tanh(7x)\,dx = \int \frac{\sinh(7x)}{\cosh(7x)}dx = \frac{1}{7}\int \frac{1}{u}\,du = \frac{1}{7}\ln|u| + C = \boxed{\frac{1}{7}\ln|\cosh(7x)| + C}$ ✓

$u=\cosh(7x)$

$du = 7\sinh(7x)\,dx$

$\frac{1}{7}du = \sinh(7x)\,dx$

8. $\int \sqrt{x}\ln(x^3)\,dx = \frac{2}{3}x^{3/2}\ln(x^3) - \frac{2}{3}\int x^{3/2}\cdot\frac{3}{x}dx$

$$\boxed{\begin{array}{ll} u=\ln(x^3) & dv=\sqrt{x}\,dx \\ du=\frac{1}{x^3}3x^2dx = \frac{3}{x}dx & v=\frac{2}{3}x^{3/2} \end{array}}$$

$= \frac{2}{3}x^{3/2}\ln(x^3) - 2\int \sqrt{x}\,dx$

$= \frac{2}{3}x^{3/2}\ln(x^3) - 2\cdot\frac{2}{3}x^{3/2} + C$

$= \boxed{\frac{2}{3}x^{3/2}\ln(x^3) - \frac{4}{3}x^{3/2} + C}$ ✓

109. $\int \frac{1}{(x^2+4)^{3/2}}\,dx = \int \frac{1}{\underbrace{(4\tan^2\theta+4)^{3/2}}_{(4)^{3/2}\sec^3\theta \to 8}} 2\sec^2\theta\, d\theta = \frac{1}{4}\int \frac{1}{\sec\theta}\, d\theta = \frac{1}{4}\int \cos\theta\, d\theta$

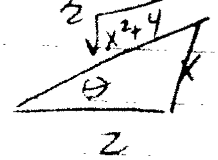
$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$= \frac{1}{4}\sin\theta + C$

$= \boxed{\frac{1}{4}\frac{x}{\sqrt{x^2+4}} + C}$ ✓

$\tan\theta = \frac{x}{2}$



110. $\int e^x \sin^2(e^x)\cos^2(e^x)\,dx = \int \sin^2 u \cos^2 u\, du = \int \left[\frac{1-\cos(2u)}{2}\right]\left[\frac{1+\cos(2u)}{2}\right] du$

$u = e^x$

$du = e^x dx$

$= \frac{1}{4}\int 1 - \cos^2(2u)\, du$

$= \frac{1}{4}\int 1 - \left[\frac{1+\cos(4u)}{2}\right] du$

$= \frac{1}{4}\cdot\frac{1}{2}\int 1 - \cos(4u)\, du$

$= \frac{1}{8}\left[u - \frac{1}{4}\sin(4u)\right] + C$

$= \boxed{\frac{1}{8}\left[e^x - \frac{1}{4}\sin(4e^x)\right] + C}$ ✓

1. $\int \frac{e^x}{\sqrt{e^{2x}+9}}\,dx = \int \frac{1}{\sqrt{u^2+9}}\,du = \frac{1}{3}\int \frac{1}{\sqrt{(\frac{u}{3})^2+1}}\,du = \int \frac{1}{\sqrt{w^2+1}}\,dw = \sinh^{-1} w + C$

$u = e^x$

$du = e^x dx$

$w = u/3$

$dw = \frac{1}{3}du$

$= \sinh^{-1}\left(\frac{u}{3}\right) + C$

$\boxed{= \sinh^{-1}\left(\frac{e^x}{3}\right) + C}$ ✓

$\int \sin^5 x \cos^2 x\, dx = \int \sin^4 x \cos^2 x \cdot \sin x\, dx = \int [\sin^2 x]^2 \cos^2 x \cdot \sin x\, dx = \int [1-\cos^2 x]^2 \cos^2 x \cdot \sin x\, dx$

$u = \cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$= -\int (1-u^2)^2 u^2\, du = -\int (1-2u^2+u^4)u^2\, du$

$= -\int u^2 - 2u^4 + u^6\, du = \int -u^2 + 2u^4 - u^6\, du = \frac{-u^3}{3} + \frac{2}{5}u^5 - \frac{u^7}{7} + C = \boxed{\frac{-\cos^3 x}{3} + \frac{2}{5}\cos^5 x - \frac{\cos^7 x}{7} + C}$ ✓

$$\int \sin^2 x\cos^3 x\,dx = \int \sin^2 x \underbrace{\cos^2 x}_{1-\sin^2 x} \cos x\,dx = \int \sin^2 x\,(1-\sin^2 x)\cos x\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$= \int u^2(1-u^2)\,du = \int u^2 - u^4\,du = \frac{u^3}{3} - \frac{u^5}{5} + C = \boxed{\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C}$$

$$\int e^x \cosh(2-e^x)\,dx = -\int \cosh u\,du = -\sinh u + C = \boxed{-\sinh(2-e^x)+C}$$

$u = 2-e^x$

$du = -e^x dx$

$-du = e^x dx$

$$\int \sec^6 x \tan^2 x\,dx = \int \underbrace{\sec^4 x}_{(\sec^2)^2} \tan^2 x \sec^2 x\,dx = \int (1+\tan^2 x)^2 \tan^2 x \sec^2 x\,dx$$

$u = \tan x$

$du = \sec^2 x\,dx$

$$= \int (1+u^2)^2 u^2\,du = \int (1+2u^2+u^4)u^2\,du = \int u^2 + 2u^4 + u^6\,du$$

$$= \frac{u^3}{3} + \frac{2u^5}{5} + \frac{u^7}{7} + C = \boxed{\frac{\tan^3 x}{3} + \frac{2\tan^5 x}{5} + \frac{\tan^7 x}{7} + C}$$

$$\sin^2 x \tan^2 x\,dx = \int \sin^2 x \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{\sin^4 x}{\cos^2 x}\,dx = \int \frac{(\sin^2 x)^2}{\cos^2 x}\,dx = \int \frac{(1-\cos^2 x)^2}{\cos^2 x}\,dx$$

$\cancel{\text{in}^2 x(\sec^2 x - 1)\,dx}$

$\cancel{\text{n}^2 \frac{1}{\cos^2} - \int \sin^2 x}$

$\cancel{\frac{\sin^2 x}{\cos^2 x}}$

$$= \int \frac{1 - 2\cos^2 x + \cos^4 x}{\cos^2 x}\,dx$$

$$= \int \frac{1}{\cos^2 x} - 2 + \cos^2 x\,dx$$

$$= \int \sec^2 x - 2 + \cos^2 x\,dx$$

$$\boxed{= \tan x - 2x + \tfrac{1}{2}\left(x + \tfrac{1}{2}\sin(2x)\right) + C}$$

$$\boxed{\tan x - \frac{3x}{2} + \frac{1}{4}\sin(2x) + C}$$

$$\int \cos^2 x\,dx = \frac{1}{2}\int 1+\cos 2x\,dx$$

$$= \frac{1}{2}\left(x + \frac{1}{2}\sin(2x)\right) + C$$

$$\frac{x}{\cosh^2 x}\,dx = \int \frac{1}{\sqrt{16-u^2}}\,du = \frac{1}{4}\int \frac{1}{\sqrt{1-\left(\frac{u}{4}\right)^2}}\,du$$

$w = \frac{u}{4}$

$dw = \frac{1}{4}du$

dx

$$dw = \arcsin w + C = \arcsin\left(\frac{u}{4}\right) + C = \boxed{\arcsin\left(\frac{\cosh x}{4}\right) + C}$$

118. $\displaystyle\int_0^1 x\tan^{-1}(x^2)\,dx = \frac{x^2}{2}\tan^{-1}(x^2)\Big|_0^1 - \int_0^1 \frac{x^2}{\cancel{2}}\cdot\frac{\cancel{2}x}{1+x^4}\,dx$

$$\boxed{\begin{array}{ll} u=\tan^{-1}(x^2) & dv = x\,dx \\ du = \dfrac{1}{1+(x^2)^2}\,2x\,dx & v=\dfrac{x^2}{2} \end{array}}$$

$\displaystyle = \frac{x^2}{2}\tan^{-1}(x^2)\Big|_0^1 - \int_0^1 \frac{x^3}{1+x^4}\,dx \quad = \frac{x^2}{2}\tan^{-1}(x^2) - \frac{1}{4}\int\frac{1}{w}\,dw$

$w = 1+x^4$

$dw = 4x^3\,dx \qquad = \frac{x^2}{2}\tan^{-1}(x^2) - \frac{1}{4}\ln|w|$

$\frac{1}{4}dw = x^3\,dx$

$\displaystyle = \frac{x^2}{2}\tan^{-1}(x^2) - \frac{1}{4}\ln|1+x^4|\Big|_0^1$ ✓ evaluate

$\left(\frac{1}{2}\tan^{-1}(1) - \frac{1}{4}\ln(2)\right) - \left(0 + \cancel{\tan^{-1}0}^{\,0} - \frac{1}{4}\cancel{\ln 1}^{\,0}\right) = \frac{1}{2}\frac{\pi}{4} - \frac{\ln 2}{4}$ $\boxed{\frac{\pi}{8} \ldots}$

119. $\displaystyle\int \tan^5 x\sec^2 x\,dx = \int \tan^4 x\sec^2 x\sec x\tan x\,dx$

$\displaystyle = \int (\tan^2 x)^2\sec^2 x\sec x\tan x\,dx = \int(\sec^2 x-1)^2\sec^2 x\sec x\tan x\,dx = \int(u^2-1)^2u^2\,du = \int(u^4-2u^2+1)u^2\,du$

$u = \sec x$

$du = \sec x\tan x\,dx \qquad \displaystyle = \int u^6 - 2u^4 + u^2\,du = \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} + C = \boxed{\frac{\sec^7 x}{7} - \frac{2\sec^5 x}{5} + \frac{\sec^3 x}{3} + \ldots}$ ✓

120. $\displaystyle\int \frac{x^2}{x^6+1}\,dx = \int\frac{x^2}{(x^3)^2+1}\,dx = \frac{1}{3}\int\frac{1}{u^2+1}\,du = \frac{1}{3}\arctan u + C = \boxed{\frac{1}{3}\arctan(x^3)+C}$ ✓

$u = x^3$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

121. $\displaystyle\int_1^{e^2} x\ln\sqrt{x}\,dx = \frac{x^2}{2}\ln\sqrt{x}\Big|_1^{e^2} - \int_1^{e^2}\frac{x^{\cancel{2}}}{2}\,\frac{1}{2\cancel{x}}\,dx = \frac{x^2}{2}\ln\sqrt{x}\Big|_1^{e^2} - \frac{1}{4}\int_1^{e^2} x\,dx$ ✓

$$\boxed{\begin{array}{ll} u=\ln\sqrt{x} & dv = x\,dx \\ du = \dfrac{1}{\sqrt{x}}\cdot\dfrac{1}{2\sqrt{x}}\,dx & v = \dfrac{x^2}{2} \\ \quad = \dfrac{1}{2x}\,dx & \end{array}}$$

$\displaystyle = \frac{x^2}{2}\ln\sqrt{x}\Big|_1^{e^2} - \frac{x^2}{8}\Big|_1^{e^2} = \frac{e^4}{2}\cancel{\ln\sqrt{e^2}}^{\,1}_{\ln e} - \cancel{\frac{1}{2}\ln\sqrt{1}}^{\,0} - \left[\frac{e^4}{8} - \frac{1}{8}\right]$

$\displaystyle = \frac{e^4}{2} - \frac{e^4}{8} + \frac{1}{8} = \boxed{\frac{3e^4}{8} + \frac{1}{8}}$ ✓

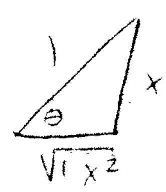
122. $\displaystyle\int\frac{x^2}{(1-x^2)^{3/2}}\,dx = \int\frac{\sin^2\theta}{\underbrace{(1-\sin^2\theta)^{3/2}}_{\cos^3\theta}}\cos\theta\,d\theta$

$x = \sin\theta$

$dx = \cos\theta\,d\theta$

$\displaystyle = \int\frac{\sin^2\theta}{\cos^2\theta}\,d\theta = \int\tan^2\theta\,d\theta = \int\sec^2\theta - 1\,d\theta = \tan\theta - \theta + C$

$\displaystyle = \boxed{\frac{x}{\sqrt{1-x^2}} - \arcsin x + C}$ ✓



123. $\displaystyle\int_1^e (\ln x)^2\,dx = x[\ln x]^2\Big|_1^e - \int_1^e \cancel{x}\,\frac{2\ln x}{\cancel{x}}\,dx = x\ln^2(x)\Big|_1^e - 2\int_1^e \ln x\,dx$

$\boxed{\begin{array}{ll} u=(\ln x)^2 & dv = dx \\ du = \frac{2\ln x}{x}\,dx & v = x \end{array}}$ $\boxed{\begin{array}{ll} u=\ln x & dv= \\ du=\frac{1}{x}dx & v= \end{array}}$

$= x\ln^2 x\Big|_1^e - 2\left[x\ln x\Big|_1^e - \int_1^e x\cdot\frac{1}{x}\,dx\right]$

$= x\ln^2 x\Big|_1^e - 2x\ln x\Big|_1^e + 2x\Big|_1^e$

$= e\ln^2 e - 1\ln^2 1 - 2e\ln e + 2\ln 1 + 2e - 2$

$= e - 0 - 2e + 2e - 2 = \boxed{e-2}$

124. $\displaystyle\int_0^{\sqrt3} \frac{1}{\sqrt{4-x^2}} + \frac{1}{x^2+9}\,dx = \int_0^{\sqrt3}\frac{1}{\sqrt{4-x^2}}\,dx + \int_0^{\sqrt3}\frac{1}{x^2+9}\,dx$

$\displaystyle= \frac12\int_0^{\sqrt3}\frac{1}{\sqrt{1-(\frac{x}{2})^2}}\,dx + \frac19\int_0^{\sqrt3}\frac{1}{(\frac{x}{3})^2+1}\,dx$

$u = x/2$, $du = \frac12 dx$, $2du = dx$ $\quad w = x/3$, $dw = 1/3\,dx$, $3dw = dx$

$\displaystyle= \int_0^{\sqrt3/2}\frac{1}{\sqrt{1-u^2}}\,du + \frac39\int_0^{1/\sqrt3}\frac{1}{w^2+1}\,dw$

$\boxed{\begin{array}{l} x=0 \Rightarrow u=0 \\ x=\sqrt3 \Rightarrow u=\sqrt3/2 \end{array}}$ $\quad x=0\Rightarrow w=0$, $x=\sqrt3 \Rightarrow w=\sqrt3/3=\frac{1}{\sqrt3}$

$\arcsin\frac{x}{2} + \frac13\arctan\frac{x}{3}$

$\downarrow$

$\pi/3$

$= \arcsin u\Big|_0^{\sqrt3/2} + \frac13\arctan w\Big|_0^{1/\sqrt3}$

$\left[\arcsin\frac{\sqrt3}{2} - \arcsin 0\right] + \frac13\left[\arctan\frac{1}{\sqrt3} - \arctan 0\right]$

$\frac{\pi}{3} + \frac13\cdot\frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{18} = \frac{6\pi}{18} + \frac{\pi}{18} = \boxed{\frac{7\pi}{18}}$

125. SWITCH TO 2X

$\displaystyle\int_{\pi/12}^{\pi/6} x\cos(2x)\,dx = \frac{x\sin(2x)}{2}\Big|_{\pi/12}^{\pi/6} - \frac12\int_{\pi/12}^{\pi/6}\sin(2x)\,dx = \frac{x\sin(2x)}{2} + \frac14\cos(2x)\Big|_{\pi/12}^{\pi/6}$

$\boxed{\begin{array}{ll} u=x & dv=\cos(2x)dx \\ du=dx & v=\frac12\sin(2x) \end{array}}$

$= \left[\frac12\cdot\frac{\pi}{6}\sin\left(\frac{\pi}{3}\right) + \frac14\cos\left(\frac{\pi}{3}\right)\right] - \left[\frac12\cdot\frac{\pi}{12}\sin\left(\frac{\pi}{6}\right) + \frac14\cos\left(\frac{\pi}{6}\right)\right]$

$= \frac{\pi}{12}\cdot\frac{\sqrt3}{2} + \frac14\cdot\frac12 - \frac{\pi}{24}\cdot\frac12 - \frac14\cdot\frac{\sqrt3}{2}$

$= \boxed{\frac{\sqrt3\pi}{24} + \frac18 - \frac{\pi}{48} - \frac{\sqrt3}{8}}$

$\displaystyle\int\frac{x^4}{\sqrt{9x^{10}+1}}\,dx = \int\frac{x^4}{\sqrt{(3x^5)^2+1}}\,dx = \frac{1}{15}\int\frac{1}{\sqrt{w^2+1}}\,dw$

$w = 3x^5$
$dw = 15x^4dx$
$\frac{1}{15}dw = x^4dx$

$= \frac{1}{15}\sinh^{-1}w + C = \boxed{\frac{1}{15}\sinh^{-1}(3x^5)+C}$

$\displaystyle\int x^{13}\sqrt{x^7+1}\,dx = \int x^7x^6\sqrt{x^7+1}\,dx = \frac17\int(u-1)\sqrt{u}\,du = \frac17\int u^{3/2} - u^{1/2}\,du = \frac17\left[\frac25u^{5/2} - \frac23u^{3/2}\right] + C$

$u = x^7+1 \Rightarrow x^7 = u-1$
$du = 7x^6dx$

$= \boxed{\frac{2}{35}(x^7+1)^{5/2} - \frac{2}{21}(x^7+1)^{3/2} + C}$

128. $\int x^5 e^{x^2}\,dx = \int x^4 x\, e^{x^2}\,dx = \frac{1}{2}\int w^2 e^w\,dw$ (with $x^4 = (x^2)^2$)

Double Integration By Parts

$w = x^2$
$dw = 2x\,dx$
$\frac{1}{2}dw = x\,dx$

$\boxed{u = w^2 \quad dv = e^w dw \\ du = 2w\,dw \quad v = e^w}$

$= \frac{1}{2}\left[w^2 e^w - 2\int w e^w\,dw\right]$

$\boxed{u = w \quad dv = e^w dw \\ du = dw \quad v = e^w}$

$= \frac{1}{2}\left[w^2 e^w - 2\left\{w e^w - \int e^w\,dw\right\}\right]$

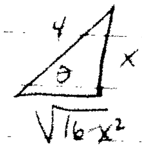
$= \boxed{\frac{1}{2} w^2 e^w - w e^w + e^w + C}$

$= \boxed{\frac{1}{2} x^4 e^{x^2} - x^2 e^{x^2} + e^{x^2} + C}$

129. $\int \frac{x^2}{\sqrt{16 - x^2}}\,dx = \int \frac{16\sin^2\theta}{\sqrt{16 - 16\sin^2\theta}}\, 4\cos\theta\,d\theta$

$x = 4\sin\theta$
$dx = 4\cos\theta$

$= \int \frac{16\sin^2\theta\, \cancel{4\cos\theta}\,d\theta}{\cancel{4\cos\theta}} = 16\int \sin^2\theta\,d\theta = 16\cdot\frac{1}{2}\int 1 - \cos 2\theta\,d\theta = 8\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$



$= 8\left[\arcsin\frac{x}{4} - \frac{1}{2}\, 2\sin\theta\cos\theta\right] + C$

$\boxed{= 8\left[\arcsin\left(\frac{x}{4}\right) - \frac{x}{4}\cdot\frac{\sqrt{16 - x^2}}{4}\right] + C}$

$8\arcsin\left(\frac{x}{4}\right) - \frac{x\sqrt{16 - x^2}}{2} + C$

130. $\int x\sqrt{x+1}\,dx = \int (u-1)\sqrt{u}\,du = \int u^{3/2} - u^{1/2}\,du = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C}$

$u = x + 1 \Rightarrow x = u - 1$
$du = dx$

31. $\int \frac{x^7}{(7 - x^4)^{3/2}}\,dx = \int \frac{x^4 x^3}{(7 - x^4)^{3/2}}\,dx = -\frac{1}{4}\int \frac{7 - u}{u^{3/2}}\,du = -\frac{1}{4}\int 7u^{-3/2} - u^{-1/2}\,du$

$u = 7 - x^4 \Rightarrow x^4 = 7 - u$
$du = -4x^3\,dx$
$-\frac{1}{4}du = x^3\,dx$

$= -\frac{1}{4}\left[7(-2)u^{-1/2} - 2u^{1/2}\right] + C$

$= \boxed{\frac{7}{2\sqrt{7 - x^4}} + \frac{1}{2}\sqrt{7 - x^4} + C} = \frac{7 + 7 - x^4}{2\sqrt{7 - x^4}} = \frac{14 - x^4}{2\sqrt{7 - x^4}}$

2. $\int x^3\sqrt{9 - x^2}\,dx = \int x^2 x\sqrt{9 - x^2}\,dx = -\frac{1}{2}\int (9 - u)\sqrt{u}\,du = -\frac{1}{2}\int 9u^{1/2} - u^{3/2}\,du = -\frac{1}{2}\left[9\cdot\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right] + C$

$u = 9 - x^2 \Rightarrow x^2 = 9 - u$
$du = -2x\,dx$
$-\frac{1}{2}du = x\,dx$

$= -\frac{1}{2}\left[6(9 - x^2)^{3/2} - \frac{2}{5}(9 - x^2)^{5/2}\right] + C$

$= \boxed{-3(9 - x^2)^{3/2} + \frac{1}{5}(9 - x^2)^{5/2} + C}$

I.B.P. $= \int x^2\, x\sqrt{9 - x^2}\,dx = x^2\left(-\frac{1}{3}(9 - x^2)^{3/2}\right) - \int \left(-\frac{1}{3}\right)(9 - x^2)^{3/2}\cdot 2x\,dx$

$\boxed{u = x^2 \quad dv = x\sqrt{9 - x^2}\,dx \\ du = 2x\,dx \quad v = -\frac{1}{2}\cdot\frac{2}{3}(9 - x^2)^{3/2}}$

$= -\frac{1}{3}x^2(9 - x^2)^{3/2} + \frac{2}{3}\cdot\frac{2}{5}\left(-\frac{1}{2}\right)(9 - x^2)^{5/2} + C$

$= (9 - x^2)^{3/2}\left[-\frac{1}{3}x^2 - \frac{2}{15}(9 - x^2)\right] + C$

$= (9 - x^2)^{3/2}\left[-\frac{1}{3}x^2 - \frac{6}{5} + \frac{2}{15}x^2\right] + C$

$= (9 - x^2)^{3/2}\left[-\frac{3}{15}x^2 - \frac{6}{5}\right] + C$

$= (9 - x^2)^{3/2}\left[-\frac{1}{5}x^2 - \frac{6}{5}\right] + C$

$= (9 - x^2)^{3/2}\left[-3 + \frac{9}{5} - \frac{1}{5}x^2\right] + C$

$(9 - x^2)^{3/2}\left[-3 + \frac{1}{5}(9 - x^2)\right] + C$

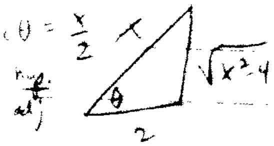
133. $\displaystyle\int \sqrt{\frac{x^2-4}{x}}\,dx = \int \frac{\overbrace{\sqrt{4\sec^2\theta-4}}^{2\tan\theta}}{\cancel{2\sec\theta}}\cdot \cancel{2\sec\theta}\tan\theta\,d\theta = 2\int \tan^2\theta\,d\theta = 2\int \sec^2\theta - 1\,d\theta$

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$= 2\left[\tan\theta - \theta\right] + C$

$= \boxed{2\left[\frac{\sqrt{x^2-4}}{2} - \operatorname{arcsec}\left(\frac{x}{2}\right)\right] + C}$



or $\Rightarrow \tan\theta = \frac{\sqrt{x^2-4}}{2}$

134. $\displaystyle\int \frac{x^2}{x^2+3}\,dx = \int \frac{x^2+3-3}{x^2+3}\,dx = \int \cancel{\frac{x^2+3}{x^2+3}}^{1} - 3\int \frac{1}{x^2+3}\,dx = x - \cancel{3}\int \frac{1}{\cancel{3}\left[\left(\frac{x}{\sqrt{3}}\right)^2+1\right]}\,dx = x - \sqrt{3}\int \frac{1}{w^2+1}\,dw$

$w = \frac{x}{\sqrt{3}}$

$dw = \frac{1}{\sqrt{3}}\,dx$

$\sqrt{3}\,dw = dx$

$= x - \sqrt{3}\arctan w + C$

$= \boxed{x - \sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right) + C}$

or try trig sub.

$x = \sqrt{3}\tan\theta$

$dx = \sqrt{3}\sec^2\theta\,d\theta$

$\displaystyle\int \frac{x^2}{x^2+3}\,dx = \int \frac{3\tan^2\theta}{\underbrace{3\tan^2\theta + 3}_{\cancel{3\sec^2\theta}}}\sqrt{3}\cancel{\sec^2\theta}\,d\theta$

$= \sqrt{3}\displaystyle\int \tan^2\theta\,d\theta = \sqrt{3}\int \sec^2\theta - 1\,d\theta = \sqrt{3}\left[\tan\theta - \theta\right] + C = \sqrt{3}\left[\frac{x}{\sqrt{3}} - \arctan\left(\frac{x}{\sqrt{3}}\right) + C\right]$

135. $\displaystyle\int_{-3}^{3} \sqrt{9-x^2}\,dx = \int_{\theta=-\pi/2}^{\theta=\pi/2} \sqrt{9-9\sin^2\theta}\;3\cos\theta\,d\theta = \int_{-\pi/2}^{\pi/2} 3\cos\theta\cdot 3\cos\theta\,d\theta = 9\int_{-\pi/2}^{\pi/2}\cos^2\theta\,d\theta = \frac{9}{2}\int_{-\pi/2}^{\pi/2} 1+\cos 2\theta\,d\theta$

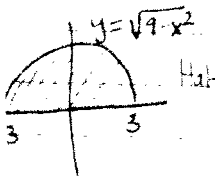
$x = 3\sin\theta$ $\quad x=-3 \Rightarrow \theta = \arcsin(-1) = -\pi/2$

$dx = 3\cos\theta\,d\theta$ $\quad x = 3 \Rightarrow \theta = \arcsin(1) = \pi/2$

$= \frac{9}{2}\left[\theta + \frac{1}{2}\sin(2\theta)\right]\Big|_{-\pi/2}^{\pi/2}$

$= \frac{9}{2}\left[\left(\frac{\pi}{2} + \cancel{\frac{1}{2}\sin\pi}^{0}\right) - \left(-\frac{\pi}{2} + \cancel{\frac{1}{2}\sin(-\pi)}^{0}\right)\right]$

$= \frac{9}{2}\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \boxed{\frac{9\pi}{2}}$ ✓



Half Circle Radius 3

$\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(9) = \boxed{\frac{9\pi}{2}}$ ✓

136. $\displaystyle\int \sqrt{1-4x^2}\,dx = \int \sqrt{1-(2x)^2}\,dx = \frac{1}{2}\int \sqrt{1-w^2}\,dw = \frac{1}{2}\int \sqrt{1-\sin^2\theta}\cdot\cos\theta\,d\theta = \frac{1}{2}\int \cos\theta\cos\theta\,d\theta = \frac{1}{2}\int \cos^2\theta\,d\theta$

$w = 2x$

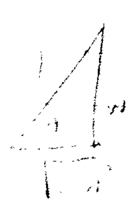
$dw = 2dx$

$\frac{1}{2}dw = dx$

$w = \sin\theta$

$dw = \cos\theta\,d\theta$

$= \frac{1}{2}\cdot\frac{1}{2}\displaystyle\int 1+\cos(2\theta)\,d\theta = \frac{1}{4}\left[\theta + \frac{1}{2}\sin(2\theta)\right] + C$



$= \frac{1}{4}\left[\arcsin w + \frac{1}{2}\,2\sin\theta\cos\theta\right] + C$

$\frac{1}{4}\left[\arcsin w + \ldots 2x\sqrt{1-w^2}\right] + C$

$\boxed{\ldots + \frac{x}{2}\sqrt{1-4x^2} + C}$

137. $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx = \int \frac{1}{4\tan^2\theta\,\underbrace{\sqrt{4\tan^2\theta+4}}_{\cancel{2}\sec\theta}}\,2\sec^2\theta\,d\theta = \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{4}\int \frac{1}{\cancel{\cos\theta}}\,\frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\,d\theta$

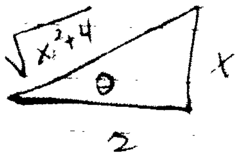
$x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$\displaystyle = \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta = \frac{1}{4}\int \frac{1}{u^2}\,du = \frac{-1}{4u} + C$

$u = \sin\theta$

$du = \cos\theta\,d\theta$



$\displaystyle = \frac{-1}{4\sin\theta} + C$

$\csc\theta = \frac{\text{hyp}}{\text{opp}}$

$\displaystyle = -\frac{1}{4}\csc\theta + C$

$\displaystyle \boxed{= -\frac{1}{4}\left[\frac{\sqrt{x^2+4}}{x}\right] + C}$ ✓

138. $\displaystyle\int \sinh^{-1}x\,dx = x\sinh^{-1}x - \int \frac{x}{\sqrt{x^2+1}}\,dx$ check

$\boxed{u = \sinh^{-1}x \quad dv = dx \quad du = \frac{1}{\sqrt{x^2+1}} \quad v = x}$

$w = x^2+1$

$dw = 2x\,dx \quad \frac{1}{2}dw = x\,dx$

$\displaystyle = x\sinh^{-1}x - \frac{1}{2}\int \frac{1}{\sqrt{w}}\,dw$

$\displaystyle = x\sinh^{-1}x - \frac{1}{\cancel{2}}\cdot\cancel{2}w^{1/2} + C = \boxed{x\sinh^{-1}x - \sqrt{x^2+1} + C}$ ✓

139. $\displaystyle\int_0^{\frac{\ln 7}{2}} \sinh(2x)\,dx = \frac{1}{2}\int_0^{\ln 7} \sinh w\,dw = \frac{1}{2}\cosh w\Big|_0^{\ln 7} = \frac{1}{2}\cosh\ln 7 - \frac{1}{2}\cancel{\cosh 0}^{\,1}$

$w = 2x \qquad x = 0 \Rightarrow w = 0$

$dw = 2dx \qquad x = \frac{\ln 7}{2} \Rightarrow w = \ln 7$

$\frac{1}{2}dw = dx$

$\displaystyle = \frac{1}{2}\left[\frac{e^{\ln 7} + \frac{1}{e^{\ln 7}}}{2}\right] - \frac{1}{2} = \frac{1}{4}\left[7 + \frac{1}{7}\right] - \frac{1}{2}$

$\displaystyle = \frac{1}{4}\left[\frac{49}{7} + \frac{1}{7}\right] - \frac{1}{2}$

$\displaystyle = \frac{1}{4}\left(\frac{50}{7}\right) - \frac{1}{2} = \frac{25}{14} - \frac{7}{14}$

$\displaystyle = \frac{18}{14} = \boxed{\frac{9}{7}}$ ✓

0. $\displaystyle\int (e^x + \cos x)^2\,dx = \int e^{2x} + \underbrace{2e^x\cos x}_{(*)} + \underbrace{\cos^2 x}_{(**)}\,dx$

) $\displaystyle = \int e^x\cos x\,dx = e^x\sin x - \int e^x\sin x\,dx = e^x\sin x - \left[-e^x\cos x - \int -e^x\cos x\,dx\right]$

$\boxed{u = e^x \quad dv = \cos x\,dx \quad du = e^x\,dx \quad v = \sin x}$

$\boxed{u = e^x \quad dv = \sin x\,dx \quad du = e^x\,dx \quad v = -\cos x}$

Combine: $\displaystyle\int e^x\cos x\,dx = e^x\sin x + e^x\cos x - \int e^x\cos x\,dx$

$\displaystyle 2\int e^x\cos x\,dx = e^x\sin x + e^x\cos x$

don't need to divide out by 2 → that's what we have

$\displaystyle \text{s}^2x\,dx = \frac{1}{2}\int (1 + \cos(2x))\,dx = \frac{1}{2}\left[x + \frac{1}{2}\sin(2x)\right] + C$

can leave

$\displaystyle = \frac{1}{2}\left[x + \cancel{\tfrac{1}{2}}\cdot\cancel{2}\sin x\cos x\right] + C$

gether: $\displaystyle\boxed{= \frac{1}{2}e^{2x} + (e^x\sin x + e^x\cos x) + \frac{1}{2}\left[x + \frac{1}{2}\sin(2x)\right] + C}$ ✓

141. $\int_1^e \sqrt{x}\ln x\,dx = (\ln x)\tfrac{2}{3}x^{3/2}\Big|_1^e - \int_1^e \tfrac{2}{3}x^{1/2}\left(\tfrac{1}{x}\right)dx = (\ln x)\tfrac{2}{3}x^{3/2}\Big|_1^e - \tfrac{2}{3}\int_1^e x^{1/2}\,dx$

$\boxed{\begin{array}{ll} u=\ln x & dv=\sqrt{x}\,dx \\ du=\frac{1}{x}dx & v=\frac{2}{3}x^{3/2} \end{array}}$

$= (\ln x)\tfrac{2}{3}x^{3/2} - \tfrac{2}{3}\cdot\tfrac{2}{3}x^{3/2}\Big|_1^e$

$= \left[\cancel{\ln e}^{\,1}\,\tfrac{2}{3}e^{3/2} - \tfrac{4}{9}e^{3/2}\right] - \left[\cancel{(\ln 1)\tfrac{2}{3}(1)^{3/2}}^{\,0} - \tfrac{4}{9}(1)\right]$

$= \tfrac{2}{3}e^{3/2} - \tfrac{4}{9}e^{3/2} + \tfrac{4}{9}$

$= \tfrac{6}{9}e^{3/2} - \tfrac{4}{9}e^{3/2} + \tfrac{4}{9} = \boxed{\tfrac{2}{9}e^{3/2} + \tfrac{4}{9}}$ ✓

142. $\displaystyle\int \frac{(e^x-1)e^x}{e^{2x}+1}\,dx = \int \frac{u-1}{u^2+1}\,du = \int \frac{u}{u^2+1} - \frac{1}{u^2+1}\,du$

$u=e^x$
$du=e^x dx$

$w=u^2+1$
$\tfrac{1}{2}dw = u\,du$

$= \tfrac{1}{2}\int \frac{1}{w}dw - \arctan u = \tfrac{1}{2}\ln|w| - \arctan u + C$

$= \tfrac{1}{2}\ln|u^2+1| - \arctan u + C$

$= \boxed{\tfrac{1}{2}\ln[e^{2x}+1) - \arctan e^x + C}$ ✓

43. $\displaystyle\int \frac{\sin^3 x}{\sqrt{\cos x}}\,dx = \int \frac{\sin^2 x}{\sqrt{\cos x}}\sin x\,dx = \int \frac{1-\cos^2 x}{\sqrt{\cos x}}\sin x\,dx$

$u=\cos x$
$du=-\sin x\,dx$
$-du=\sin x\,dx$

$= -\int \frac{1-u^2}{\sqrt{u}}\,du = -\int u^{-1/2} - u^{3/2}\,du$

$= -\left[2u^{1/2} - \tfrac{2}{5}u^{5/2}\right] + C$ ✓

$= \boxed{-2\sqrt{\cos x} + \tfrac{2}{5}(\cos x)^{5/2} + C}$

44. $\displaystyle\int \frac{x+3}{\sqrt{4-x^2}}\,dx = \int \frac{x}{\sqrt{4-x^2}}\,dx + \int \frac{3}{\sqrt{4-x^2}}\,dx = -\tfrac{1}{2}\int \frac{1}{\sqrt{w}}\,dw + \tfrac{3}{\cancel{2}}\int \frac{1}{\sqrt{1-u^2}}\cancel{2}\,du$

$w=4-x^2$
$dw=-2x\,dx$
$-\tfrac{1}{2}dw = x\,dx$

$2\sqrt{1-\left(\tfrac{x}{2}\right)^2}$
$u=\tfrac{x}{2}$
$du=\tfrac{1}{2}dx$

$= -\tfrac{1}{\cancel{2}}\cancel{2}w^{1/2} + 3\arcsin\left(\tfrac{x}{2}\right) + C$

$= \boxed{-\sqrt{4-x^2} + 3\arcsin\left(\tfrac{x}{2}\right) + C}$ ✓

i. $\displaystyle\int \sin(\ln x)\,dx = \int e^w \sin w\,dw = \boxed{+\tfrac{1}{2}\left[-e^w\cos w + e^w \sin w\right] + C} = \tfrac{1}{2}\left[-\cancel{e^{\ln x}}^{\,x}\cos\ln x + \cancel{e^{\ln x}}^{\,x}\sin\ln x\right] + C$

$(*)$

$w=\ln x \Rightarrow x=e^w$
$dw=\tfrac{1}{x}dx$
$\Rightarrow e^w dw = dx$

$= \boxed{\tfrac{1}{2}\left[-x\cos\ln x + x\sin\ln x\right] + C}$ ✓

$(*)$ $\displaystyle\int e^w \sin w\,dw = -e^w\cos w + \int e^w\cos w\,dw = -e^w\cos w + e^w\sin w - \int e^w \sin w\,dw$

$\boxed{\begin{array}{ll} u=e^w & dv=\sin w\,dw \\ du=e^w dw & v=-\cos w \end{array}}$

$\boxed{\begin{array}{ll} u=e^w & dv=\cos w\,dw \\ du=e^w dw & v=\sin w \end{array}}$

$\Rightarrow 2\int e^w \sin w\,dw = -e^w\cos w + e^w\sin w$

$\Rightarrow \int e^w \sin w\,dw = \tfrac{1}{2}[-e^w\cos w + e^w\sin w]$

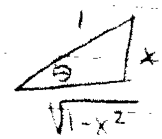
146. $\int x \arcsin x \, dx \quad = \frac{x^2}{2}\arcsin x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

$$\boxed{\begin{array}{ll} u = \arcsin x & dv = x \\ du = \frac{1}{\sqrt{1-x^2}}\,dx & v = \frac{x^2}{2} \end{array}}$$

$\underbrace{\begin{array}{l} x = \sin\theta \\ dx = \cos\theta \, d\theta \end{array}}$

$-\frac{1}{2}\left[\int \frac{\sin^2\theta}{\cancel{\cos\theta}}\,\cancel{\cos\theta}\,d\theta\right]$ ($\cos\theta = \sqrt{1-\sin^2\theta}$)

$-\frac{1}{2}\int \sin^2\theta \, d\theta$



$-\frac{1}{2}\cdot\frac{1}{2}\int 1 - \cos(2\theta)\,d\theta$

$-\frac{1}{4}\left[\theta - \frac{1}{2}\sin(2\theta)\right] + C$

$-\frac{1}{2}\,2\sin\theta\cos\theta + C$

$$= \boxed{\frac{x^2}{2}\arcsin x - \frac{1}{4}\left[\arcsin x - x\sqrt{1-x^2}\right] + C}$$ ✓

147. $\int (\arcsin x)^2 \, dx \quad = x(\arcsin x)^2 - 2\int \frac{x \arcsin x}{\sqrt{1-x^2}}\,dx = x(\arcsin x)^2 - 2\int w \sin w \, dw$

$$\boxed{\begin{array}{ll} u = (\arcsin x)^2 & dv = dx \\ du = \frac{2\arcsin x}{\sqrt{1-x^2}}\,dx & v = x \end{array}}$$

$w = \arcsin x \Rightarrow x = \sin w$

$dw = \frac{1}{\sqrt{1-x^2}}\,dx$

$$\boxed{\begin{array}{ll} u = w & dv = \sin w \, dw \\ du = dw & v = -\cos w \end{array}}$$

$-2\left[-w\cos w - \int(-\cos w)\,dw\right]$

$= \underbrace{x(\arcsin x)^2 + 2w\cos w}\; - 2\int \cos w \, dw$

$-2\sin w + C$

$$= \boxed{x(\arcsin x)^2 + 2\arcsin x \cdot \underbrace{\cos(\arcsin x)}_{\sqrt{1-x^2}} - 2x + C}$$ ✓