Math 12, Fall 2006

Final Exam, Tuesday, December 19, 2006

No books, notes, calculators, cell phones, iPods, etc.
Express yourself clearly and legibly. Explain your reasoning and show your work.
Little or no credit may be awarded if you fail to justify your answers.

There are 11 problems (with a total of 23 parts) totalling 180 points.

1. (30 points) Evaluate each of the following definite and indefinite integrals:

   (a) $\displaystyle\int_0^4 \frac{dx}{(9+x^2)^{3/2}}$ (b) $\displaystyle\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$ (c) $\displaystyle\int \frac{12}{x^2(x-2)}\,dx$

2. (16 points) Determine whether each of the following improper integrals converges; in case of convergence, evaluate the integral.

   (a) $\displaystyle\int_{-\infty}^0 \frac{dx}{1+4x^2}$ (b) $\displaystyle\int_{-1}^3 \frac{dx}{x^4}$

3. (16 points) Determine whether or not each limit exists. If it does, find its value.

   (a) $\displaystyle\lim_{x\to 0} \frac{\cosh x - 1}{x^2}$ (b) $\displaystyle\lim_{x\to 0^+} (\cos x)^{1/x^2}$

4. (16 points) Determine whether each of the following series converges or diverges.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{\sin^2(2n)}{2^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n^n}$

5. (12 points) Determine whether the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\sqrt{n}}{n+68}$ converges absolutely, converges conditionally, or diverges.

6. (14 points) The following series both converge. Find the **sum** of each.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2^{n+2}}{3^n}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{6^{2n+1}(2n+1)!}$

7. (12 points) Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n\cdot 3^n}$. That is, find the set of all real numbers x for which the series converges.

8. (16 points) Find the Maclaurin series for each of the following functions. If you use "dot-dot-dot" notation in your answers, be sure to write out at least four **nonzero** terms of each series.

   (a) $g(x) = 3xe^{x^4}$ (b) $f(x) = \dfrac{x}{(1+2x)^2}$

9. (16 points) Let R be the region in the plane beneath the graph of $y = e^x$ and above the x-axis, for $0 \leq x \leq 2$.

   (a) Find the volume of the solid formed by revolving R about the line $y = -1$.

   (b) Find the volume of the solid formed by revolving R about the line $x = 4$.

10. (16 points) Let C be the parametric curve given by $x = t + t^2$, $y = t - t^2$ for $0 \leq t \leq 2$.

   (a) Compute the slope $\dfrac{dy}{dx}$ of this curve.

   (b) Using the defining equations and your formula for the slope from part (a), sketch the curve C, and label (with rectangular coordinates) any intersection with the x- or y-axis.

   (c) Write down, but do not evaluate, an integral (in the variable t) giving the length of the curve C.

   (d) Write down, but do not evaluate, an integral (in the variable t) giving the **surface area** generated by revolving C about the x-axis.

11. (16 points) Consider the curve defined by the polar coordinates equation $r = \sin(3\theta)$.

   (a) Sketch the curve.

   (b) Compute the total area of the region or regions in the plane enclosed by the curve.