Math 12-Benedetto

What you need to know for Exam 3

You should know Sections 12.8–12.10, Sections 11.1–11.2 and Sections 6.2–6.3. The test will not explicitly cover the material from earlier sections, but of course it will still be assumed that you know how to deal with convergence of infinite series, exponentials, logarithms, inverse trig functions, L'Hôpital's rule, substitution, and so on. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- 12.8: Power Series. Know the definition, and remember that it includes not just series of the form $\sum c_n x^n$ but also series like $\sum c_n (x-a)^n$, where a is a constant, called the **center** of the power series. Know how to find the radius and interval of convergence of a given power series. Remind yourself what the 3 options are for intervals of convergence. See Theorem 3 on page 761. You will need to recall the works of Sections 12.2–12.7 in order to analyze the specific series that arises at each endpoint.

- 12.9: Power Series Representations of Functions. Know how to use power series you already know (like that for $1/(1-x)$, or (in later sections) e^x, $\cos x$, $\sin x$) to find power series for other functions by the following five operations: (1) substituting monomials like $-3x$ or $2x^3$ for x, or (2) multiplying by a polynomial, or (3) adding two power series (centered at the same center), or (4) integrating (usually followed by evaluating at $x = a$ to find the constant C from integration), or (5) differentiating. Know how each of these operations may affect the interval of convergence. See the Theorem on Page 766.

- 12.10: Taylor and MacLaurin Series. Know what Taylor (and MacLaurin) Series are, and how to compute them directly, using the derivatives of f. (Warning: make sure to plug the number a (which is 0 for MacLaurin) in for x in all the derivatives; so use $f^{(n)}(a)$, not $f^{(n)}(x)$.) In other words, know boxes 5–7, pages 771–772, and **know what they mean**. Know the box on page 779 **cold**, including the intervals of convergence, except for the binomial series. Know what Taylor and MacLaurin **polynomials** are, and know how to compute them.

- 11.1. Parametric Equations: Know what a parametric curve is, and be able to sketch such a curve (including the direction it is traversed) by plotting points, or by eliminating the parameter. (The latter is either solving for y in terms of x, or solving for x in terms of y, or if sines and cosines are floating around, you can try squaring the terms and see if the parametric equations satisfy some equation of a circle, say.)

- 11.2. Calculus with Parametric Curves: Know, and be able to use, the formulas for slopes of tangent lines (box 2, page 666) and for d^2y/dx^2 (top of page 667) for parametric curves. Know how to find the arc length of a parametric curve (Theorem 6, page 670). The square root chunk is sometimes simple and other times, you need to manipulate the algebraic computations to find a perfect square under the square root. Know how to find the surface area a parametric curve sweeps out when rotated about the x or y-axis (formula 7, page 671). The book does not state the formula for rotation about the y-axis, but I gave that in class.

- 6.2–6.3. Volumes. Know the method of disks and washers, plus the method of cylindrical shells for computing volumes of revolution. You do not need to know how to compute volumes which are not volumes of revolution. However, you do need to be able to compute volumes of revolution when the axis of rotation is any horizontal or vertical line. In theory, you should be able to decide whether you want to use shells or disks/washers for a given problem.

Some things you don't need to know

- Section 12.10: Taylor Series Remainder stuff (like Box 8 and Box 9, page 773).
- Section 12.10: Binomial Series (Box 17, page 778)
- All of Section 12.11.
- Chapter 11: Anything about graphing calculators.
- Section 11.1: All the stuff about **families** of parametric curves.
- Section 11.2: Area for Parametric Equations (bottom page 668)
- Section 11.3–11.4: Polar Coordinates (for right now anyhow...)
- Chapter 11: The specific names for various curves.
- Section 6.2: Volumes that aren't volumes of revolution.

Tips

- For power series: finding the radius of convergence is hopefully not too challenging; but be careful in the computations when using the Ratio Test. If you're asked for **interval** of convergence, don't forget to check the endpoints; at the endpoints, you will have to use convergence tests **other than** the Ratio or Root Test, since they're inconclusive there.

  Remember $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$ and therefore, $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \frac{1}{e}$.

- For power series representations: if you're asked to find the Taylor series (or power series rep) of a function $f(x)$, or even a Taylor polynomial, your first attempt should be to write f in terms of the basic functions $1/(1-x)$, e^x, $\cos x$, $\sin x$ using only the operations that are good for power series (see the 12.9 discussion above). If that doesn't seem to work, then you can try to blast out the Taylor series by taking all those derivatives. (The "blast out the Taylor series" method has easier steps, but it only works if the derivatives are fairly simple and end up fitting a nice pattern. It also takes much more time. Finding the MacLaurin series for $x^4\cos x$ takes about five seconds if you just multiply the series for $\cos x$ by x^4; but it's a horrendous and time-consuming monster to compute by blasting out all the derivatives.)

- If you're asked for the **sum** of a series, on Exam 2 that meant it'd have to be something like a geometric series or a telescoping series. Now we have a third possibility: that you can get that series by plugging in x equals some specific number in some specific power series. For example, $\sum_{n=0}^{\infty}\frac{5^{n-2}}{2^{2n}\cdot n!}$ can be rewritten as $\frac{1}{25}\sum_{n=0}^{\infty}\frac{(5/4)^n}{n!} = \frac{1}{25}e^{5/4}$. Similarly, $\sum_{n=1}^{\infty}\frac{n}{3^n} = \frac{1}{3}\sum_{n=1}^{\infty} n\left(\frac{1}{3}\right)^{n-1}$, which is 1/3 times the power series for $1/(1-x)^2$ (found by differentiating $1/(1-x)$) at $x = 1/3$; so the sum is $(1/3)\cdot[1/(1-(1/3))^2] = (1/3)\cdot(9/4) = 3/4$.

  As before, you will get no credit for verifying that a series converges if I asked you what the sum is, not whether it converges.

- Know the formulas, but try not to just memorize the Volumes of Revolution Formulas. Take the time to picture the volume for one general approximating Disk, Washer, or Cylinder. The integral will do the calculus for you, to find the total volume. :)