Math 12-Benedetto

What you need to know for Exam 2

You should know Sections 8.3–8.5, 8.8, and 12.1–12.7. The test will not explicitly cover the material from earlier sections, but of course it will still be assumed that you know how to deal with exponentials, logarithms, inverse trig functions, L'Hôpital's rule, substitution, and so on. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- 8.3: Mainly what will be covered from this section is Completing the Square.
- 8.4: Partial Fractions. Step I: Long division to reduce a rational function to a polynomial plus a proper fraction (one with deg(numer) $<$ deg(denom)). Step II: break the proper fraction into the correct pieces (using factorization of the denominator), and solve for $A, B, \ldots$. Then integrate the resulting pieces (usually, but not always, ln's (with maybe a u-subst. thrown in) and arctan's). Be familiar with all the common partial fraction decompositions.
- 8.5: Integration Strategy. This section reminds you how to approach integrating most integrals we've seen, which of course could come in any improper integral. See 8.8.
- 8.8: Improper Integrals. Be able to recognize improper integrals of either type, and know how to compute them by turning them into limits of integrals. Also know when and how to chop an integral up if there are multiple "badnesses" or a "badness" in the middle somewhere.
- 12.1: Sequences. Know what sequences are, and be able to compute their limits (or determine that they diverge). Remind yourself how to use L'Hôpital's rule, or the Squeeze Law.
- 12.2: Series. Know what series are, and **don't confuse them with sequences**. Again, know everything in this section. (Especially, but not exclusively, geometric series, and the n^{th} term divergence test.)
- 12.3: Integral Test. Know the integral test and be able to use it. Make sure you explicitly check all the requirements before applying it. Also know the p-Test.
- 12.4: Comparison Tests. Know the comparison test and the limit comparison test. Choose the testing series $\sum b_n$ wisely, and make sure to check the requirements before invoking either test.
- 12.5: Alternating Series. Know the definition of an alternating series, know how to recognize one, and know the Alternating Series Test. (And know that if you try to use AST and the terms don't go to zero, then you can't use that test; but you **can** use the divergence test in that case.)
- 12.6: Absolute Convergence, Ratio Test, (**NOT** Root Test). Know these tests (I consider Theorem 3 page 777 to be a convergence test). Know the definition of absolute convergence, conditional convergence, and divergence. Know that if you are asked whether a series converges absolutely, it **does not necessarily mean** that you should use either the Ratio or Root Test. The Ratio Test works great for series involving factorials and/or **constants** raised to the power n, even if there are other things like polynomials multiplied in.
- 12.7: Strategy for Testing Series. The main theme: get used to using the tests to decide convergence/divergence of a series even if the problem doesn't tell you which test to use.

Some things you don't need to know

- Section 8.5: I won't give you any integrals that require inverse hyperbolic functions.
- Section 8.8: Comparison Test for integrals (near the end of the section).
- Section 12.3: Proofs of the various convergence tests.
- Section 12.3: The stuff about estimating sums or remainders.
- Section 12.4 and 12.5: Estimating sums.
- Section 12.6: The Root Test or Rearrangements.
- Sections 12.4–12.6:Proofs of the various theorems and tests.

Tips

- For integrals: practice, practice, practice. Take the review packet and search the answer keys on-line. Come ask me for advice on problems you were unable to do or unsure of. On the test itself, be prepared to mess around and to try several different things on any given integeral. If you can't figure out how to do a given integral (or other problem), skip it and come back to it later. But try not to just stare; keep trying to come up with different strategies and keep writing stuff down.
- Improper Integrals: **Recognizing them**: a ∞ or $-\infty$ in the limits of integration is a dead giveaway, but also look to see whether the integrand has a vertical asymptote at either endpoint, or anywhere in between. **Computing them:** first, if needed, chop the interval up into pieces so that each piece has only one place of badness, and so that that place is at one of the endpoints. Second, compute the integral on each piece by taking a limit as t approaches the bad endpoint. The limit sign is important; don't leave it off.
- For sequences: know all the methods; but remember, when you want to use L'Hôpital Rule you must step aside and use the related functions with x as the variable. Then you can convergence about your original terms.
- Don't confuse a sequence with a series.
- For series: don't try to misuse a convergence test. If the limit of the terms is zero, the div test says nothing. (It does **not** say that the series converges, for example.) If you get stuck using a convergence test, you need to try something else. If you're really stuck, write down (briefly) what failed and why. Once you see it on paper, that might give you an idea of what will work; or if it doesn't, you may at least get some partial credit. Study the overview sheet I made with all the convergence tests listed.
- If you're asked for the **sum** of a series, that pretty much means that you must be dealing with a geometric series, or telescoping series, or the sum of two such things, or some other simple trick like that.