Math 12 Midterm Exam #3 Compact Version April 28, 2010

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [15 Points] Find the **Interval** and **Radius** of Convergence for each of the following power series. Analyze carefully and with full justification.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{(2n)!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(3x+4)^n}{n^2 4^n}$

2. [10 Points] Find the **Taylor polynomial of degree 3** for $\mathbf{f(x) = \frac{1}{x}}$ centered at $\mathbf{a = 2}$.

3. [10 Points] Find the **MacLaurin series** representation for each of the following functions. Your answer should be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$

(a) $f(x) = xe^{-x^2}$

(b) $f(x) = \dfrac{1}{1+7x}$

4. [10 Points] Use a Power Series representation for $\sin(x^2)$ to estimate the given integral within the given error. Justify in words that your error is indeed less than $\frac{1}{10}$.

$$\text{Estimate} \quad \int_0^1 \sin(x^2)\, dx \quad \text{with error less than } \frac{1}{10}$$

5. [15 Points] Find the **sum** for each of the following series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{7^n}{n!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{3\pi}{2}\right)^{2n}}{(2n+1)!}$

6. [20 Points] Volumes of Revolution

(a) Consider the region bounded by $y = \sin x$, $x = 0$, $x = \dfrac{\pi}{2}$ and the x-axis. Rotate the region about the vertical line $\boxed{x = -3}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.

(b) Consider the region bounded by $y = e^x$, $y = \ln x$, $x = 1$ and $x = 2$. Rotate the region about the horizontal line $\boxed{y = -1}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Washer Method. Sketch the solid, along with one of the approximating washers.

7. [20 Points] Consider the Parametric Curve represented by $x = 3 - 2t$ and $y = e^t + e^{-t}$.

(a) Find $\dfrac{dy}{dx}$ when $t = \ln 5$.

(b) Find the **arclength** of this parametric curve for $0 \leq t \leq 1$.

(c) Set-up, **BUT DO NOT EVALUATE!!**, the definite integral representing the **surface area** of the solid obtained by rotating this curve about the **x-axis**, for $0 \leq t \leq 1$.

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the **sum** $\displaystyle\sum_{n=0}^{\infty} \frac{n}{3^n}$

OPTIONAL BONUS #2 Compute the **sum** $\displaystyle\sum_{n=0}^{\infty} \frac{n^3}{3^n n!}$

OPTIONAL BONUS #3 Compute the sum $\displaystyle\sum_{n=0}^{\infty} \frac{n^3}{3^n}$