Name:_____

Amherst College

DEPARTMENT OF MATHEMATICS

Math 12

Midterm Exam #2

March 30, 2011

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		40
2		8
3		8
4		20
5		24
Total		100

1. [40 Points] Compute the following integral, or else show that it diverges.

(a) $\displaystyle\int_1^9 \frac{1}{(x-7)^2}\,dx$

1. (Continued) Compute the following integral, or else show that it diverges.

(b) $\displaystyle\int \frac{4x+1}{x^2-3x-10}\,dx$

1. (Continued) Compute the following integral, or else show that it diverges.

(c) $\displaystyle\int \frac{x^4 + x^2 + x + 1}{x^3 + x}\, dx$

1. (Continued) Compute the following integrals, or else show that it diverges.

(d) $\displaystyle\int_9^\infty \frac{1}{x^2 - 8x + 41}\, dx$

2. [8 Points] Determine whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do not just put down a number.

$$\left\{\left(\frac{n+1}{n}\right)^n\right\}_{n=1}^{\infty}$$

3. [8 Points] Find the **sum** of the following series (which does converge):

$$\sum_{n=1}^{\infty} \frac{(-1)^n \ 3^{n+2}}{2^{4n-1}}$$

4. [20 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n \sin^2 n}{8n^2\sqrt{n} + n + 7}$

(b) $\displaystyle\sum_{n=1}^{\infty} e^{\frac{\sin n}{n}}$

4. (Continued) Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{e^{2n} n!}$

5. [24 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\ \frac{n^2}{7^n}$

5. (Continued) In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \ \frac{n^6+5n^2+826}{n^9+7n^3+2011}$

(d) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \ \frac{1}{3^n}$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the sum of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+3n}$

OPTIONAL BONUS #2 Determine whether the following series converges or diverges.

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^{4n}}{n^7 (n!)^2 e^{8n} (2n)!}$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #3 Compute the following integral:

3. $\displaystyle\int \frac{\cos x}{\sin^3 x - 1}\, dx$

OPTIONAL BONUS #4 Compute the following integral:

4. $\displaystyle\int \frac{x^5 + x^4 + 19x^3 + 18x^3 + 18x^2 + 81x + 81}{x^4 + 18x^2 + 81}\, dx$