Math 12 Midterm Exam #2 (Compact Version) March 31, 2010

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need not simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [40 Points] Compute each of the following integrals, or else show that it diverges.

(a)
$$\int_0^9 \frac{1}{\sqrt{9-x}} dx$$
 (b) $\int \frac{5}{(x-2)(x+3)} dx$ (c) $\int \frac{x^3-1}{x^2+1} dx$

(d)
$$\int_{-\infty}^{\infty} e^x dx$$
 (e) $\int_{7}^{\infty} \frac{1}{x^2 - 6x + 25} dx$

2. [6 Points] Determine whether the following sequence converges or diverges. If it converges, compute its limit. Justify your answer. Do not just put down a number.

$$\left\{n^{\frac{1}{n}}\right\}_{n=1}^{\infty}$$

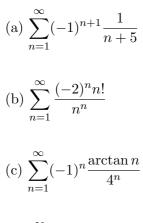
3. [8 Points] Find the **sum** of the following series (which does converge):

$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^{n-1}}{3^{2n+1}}$$

4. [18 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify that it's legal to use them. Show all of your work.

(a)
$$\sum_{n=1}^{\infty} \frac{3n^7 + 6n^{\frac{3}{2}} + 5}{8n^9 - \sqrt{n} + 441}$$
 (b) $\sum_{n=1}^{\infty} \frac{e^n}{n^2 + 1}$ (c) $\sum_{n=1}^{\infty} \left(-\frac{7}{8}\right)^n$

5. [28 Points] In each case determine whether the given series is absolutely convergent, conditionally convergent, or diverges. Justify your answers.



(d)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$$

OPTIONAL BONUS

OPTIONAL BONUS #1 Compute the sum of the following series:

$$1. \sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n^2 + 2n}\right)$$

OPTIONAL BONUS #2 Determine whether the following series converges or diverges.

2.
$$\sum_{n=1}^{\infty} \frac{(-1)^n n^{3n}}{n^3 (n!)^2 e^{n^2}}$$

OPTIONAL BONUS #3 Compute the following integral:

3.
$$\int \frac{\arctan x}{x^6} \, dx$$

OPTIONAL BONUS #4 Compute the following integral:

4.
$$\int \frac{16e^{3x}}{e^{4x} - 16} dx$$