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Amherst College

DEPARTMENT OF MATHEMATICS

Math 12

Midterm Exam #2

March 31, 2010

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		40
2		6
3		8
4		18
5		28
Total		100

1. [40 Points] Compute each of the following integrals, or else show that it diverges.

(a) $\displaystyle\int_0^9 \frac{1}{\sqrt{9-x}}\,dx$

(b) $\displaystyle\int \frac{5}{(x-2)(x+3)}\,dx$

1. (Continued) Compute each of the following integrals, or else show that it diverges.

(c) $\displaystyle\int \frac{x^3 - 1}{x^2 + 1}\, dx$

(d) $\displaystyle\int_{-\infty}^{\infty} e^x\, dx$

1. (Continued) Compute each of the following integrals, or else show that it diverges.

(e) $\displaystyle\int_7^\infty \frac{1}{x^2 - 6x + 25}\, dx$

2. [6 Points] Determine whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do not just put down a number.

$$\left\{ n^{\frac{1}{n}} \right\}_{n=1}^{\infty}$$

3. [8 Points] Find the **sum** of the following series (which does converge):

$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^{n-1}}{3^{2n+1}}$$

4. [18 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify that it's legal to use them. Show all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^7 + 6n^{\frac{3}{2}} + 5}{8n^9 - \sqrt{n} + 441}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2 + 1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(-\frac{7}{8}\right)^n$

5. [28 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n+5}$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-2)^n n!}{n^n}$

5. (Continued) In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Justify your answers.

(c) $\sum_{n=1}^{\infty}(-1)^n \frac{\arctan n}{4^n}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the sum of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n^2 + 2n}\right)$

OPTIONAL BONUS #2 Determine whether the following series converges or diverges.

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^{3n}}{n^3 (n!)^2 e^{n^2}}$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #3 Compute the following integral:

3. $\displaystyle\int \frac{\arctan x}{x^6}\,dx$

OPTIONAL BONUS #4 Compute the following integral:

4. $\displaystyle\int \frac{16e^{3x}}{e^{4x}-16}\,dx$