Review Packet for Exam #2

Math 12-D. Benedetto

Integrals: Compute each of the following integrals, or else show that it diverges.

1. $\displaystyle \int \frac{e^{4x} + 2e^{2x} - e^x}{e^{2x} + 1} \, dx$

2. $\displaystyle \int_0^3 \frac{1}{\sqrt{9 - 3x}} \, dx$

3. $\displaystyle \int_1^\infty \frac{1}{3x + 1} \, dx$

4. $\displaystyle \int_3^\infty \frac{1}{(x^2 + 16)^{\frac{3}{2}}} \, dx$

5. $\displaystyle \int_3^\infty \frac{1}{x^2 - 4x + 7} \, dx$

6. $\displaystyle \int_e^\infty \frac{1}{x(\ln x)^3} \, dx$

7. $\displaystyle \int_0^3 \frac{\arctan \sqrt{x}}{\sqrt{x}(1 + x)} \, dx$

8. $\displaystyle \int_0^\infty \frac{1}{(x + 2)(2x + 5)} \, dx$

9. $\displaystyle \int \frac{2x^2 - 2x + 6}{(x - 1)(x^2 - 2x + 7)} \, dx$

10. $\displaystyle \int_7^\infty \frac{1}{x^2 - 8x + 19} \, dx$

11. $\displaystyle \int_0^1 \frac{\ln x}{\sqrt{x}} \, dx$

12. $\displaystyle \int \frac{1}{(x + 3)(3x + 1)} \, dx$

13. $\displaystyle \int_{12}^\infty \frac{1}{x\sqrt{x - 3}} \, dx$

14. $\displaystyle \int_2^\infty \frac{1}{x^2 - 2x + 4} \, dx$

15. $\displaystyle \int \frac{1}{x^2 + 2x + 2} \, dx$

16. $\displaystyle \int_1^\infty \frac{\sqrt{x}}{1 + x^3} \, dx$

17. $\displaystyle\int_0^4 \frac{1}{(8-2x)^{\frac{1}{3}}}\ dx$

18. $\displaystyle\int \frac{1}{-x^2+2x+3}\ dx$

19. $\displaystyle\int_2^\infty \frac{1}{(x^2+4)^2}\ dx$

20. $\displaystyle\int_{-1}^1 \frac{1}{\sqrt{1-x^2}}\ dx$

21. $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\ dx$

22. $\displaystyle\int_0^1 \frac{1}{x}\ dx$

23. $\displaystyle\int_1^\infty \frac{1}{x}\ dx$

24. $\displaystyle\int_0^1 \frac{1}{x^2}\ dx$

25. $\displaystyle\int_1^\infty \frac{1}{x^2}\ dx$

26. $\displaystyle\int_0^{\frac{\pi}{2}} \tan x\ dx$

27. $\displaystyle\int_0^1 \frac{1-2x}{\sqrt{x-x^2}}\ dx$

28. $\displaystyle\int_0^\infty e^{-x}\ dx$

29. $\displaystyle\int_0^{\frac{\pi}{2}} \sec^2 x\ dx$

30. $\displaystyle\int_3^4 \frac{1}{(x-4)^2}\ dx$

31. $\displaystyle\int_1^2 \frac{1}{x\ln x}\ dx$

32. $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec x\ dx$

33. $\displaystyle\int_0^2 \frac{1}{(2x-1)^{\frac{2}{3}}}\ dx$

34. $\displaystyle\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\ dx$

35. $\displaystyle\int_1^\infty \frac{\ln x}{x}\, dx$

36. $\displaystyle\int_0^\infty \frac{1}{x+x^2}\, dx$

37. $\displaystyle\int_{-\infty}^\infty \frac{x}{(x^2+4)^{\frac{3}{2}}}\, dx$

38. $\displaystyle\int_{-4}^4 \frac{1}{(x+4)^{\frac{2}{3}}}\, dx$

39. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin x}{(\cos x)^{\frac{4}{3}}}\, dx$

40. $\displaystyle\int_{-\infty}^\infty |x|e^{-x^2}\, dx$

41. $\displaystyle\int \frac{2x-5}{x^2+2x+2}\, dx$

42. $\displaystyle\int_0^1 \frac{3x^2-1}{x^3-x}\, dx$

43. $\displaystyle\int_0^1 \frac{1}{e^x - e^{-x}}\, dx$

44. $\displaystyle\int_0^1 \frac{e^x}{\sqrt{e^x-1}}\, dx$

45. $\displaystyle\int \frac{1}{x^2+4x+5}\, dx$

46. $\displaystyle\int_0^\infty \sin^2 x\, dx$

47. $\displaystyle\int_0^1 \ln x\, dx$

48. $\displaystyle\int \frac{2x^2+3}{x(x-1)^2}\, dx$

49. $\displaystyle\int_0^1 \frac{1}{(1-x^2)^{\frac{3}{2}}}\, dx$

50. $\displaystyle\int_1^5 \frac{x}{\sqrt{x-1}}\, dx$

51. $\displaystyle\int_1^\infty \frac{1}{x(x^2+1)}\, dx$

52. $\displaystyle\int_{-\infty}^\infty x\sin x\, dx$

53. $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2 - 6x + 10}\ dx$

54. $\displaystyle\int_{-\infty}^{\infty} x\ dx$

55. $\displaystyle\int \frac{x^4 - x^3 - x - 1}{x^3 - x^2}\ dx$

56. $\displaystyle\int_{0}^{\infty} \frac{x}{e^x}\ dx$

57. $\displaystyle\int_{-5}^{0} \frac{x}{x^2 + 4x - 5}\ dx$

58. $\displaystyle\int_{-5}^{0} \frac{1}{x^2 + 4x - 5}\ dx$

59. $\displaystyle\int_{-1}^{1} \frac{1}{x^3}\ dx$

60. $\displaystyle\int \frac{x^5 + 2}{x^2 - 1}\ dx$

61. $\displaystyle\int_{0}^{6} \frac{1}{(x-2)^2}\ dx$

62. $\displaystyle\int_{0}^{\infty} \frac{1}{x^2 + 3x + 2}\ dx$

63. $\displaystyle\int_{0}^{\frac{\pi}{2}} \tan^2 x\ dx$

64. $\displaystyle\int_{0}^{2} \frac{1}{(4 - x^2)^{\frac{3}{2}}}\ dx$

65. $\displaystyle\int_{1}^{32} \frac{1}{\sqrt[5]{x - 32}}\ dx$

66. $\displaystyle\int_{-\infty}^{1} xe^{4x}\ dx$

67. $\displaystyle\int \frac{1}{(x+1)^2(x+2)}\ dx$

68. $\displaystyle\int_{0}^{1} \frac{1}{x^2\sqrt{x^2 + 16}}\ dx$

69. $\displaystyle\int \frac{4x^2 + 7x + 6}{(x+2)(x^2 + 4)}\ dx$

70. $\displaystyle\int_{1}^{\infty} \frac{1}{x(x+1)}\ dx$

4

71. $\int_{-3}^{3} \dfrac{1}{x(x+1)} \, dx$

72. $\int_{-3}^{1} \dfrac{1}{x^2 - 4} \, dx$

73. $\int_{0}^{1} \arcsin x \, dx$ (leads to improper integral)

74. $\int_{0}^{\infty} \cosh x \, dx$

75. $\int \dfrac{2x^3}{x^2 + 3} \, dx$

76. $\int \dfrac{x^2 - 1}{x^2 + 1} \, dx$

77. $\int \dfrac{\cos x (\sin^3 x + 7 \sin x + 1)}{\sin^2 x + 1} \, dx$

78. $\int \dfrac{x^2 + 5x + 2}{(x+1)(x^2+1)} \, dx$

Sequences: For each of the following sequences, decide whether it converges or diverges. If it converges, compute its limit.

79. $\left\{ \dfrac{1 + n - 7n^4}{3n^4 + 8n^3 + 9} \right\}_{n=1}^{\infty}$

80. $\left\{ \dfrac{2^n}{n!} \right\}_{n=1}^{\infty}$

81. $\left\{ \dfrac{n!}{3^n} \right\}_{n=1}^{\infty}$

82. $\left\{ \dfrac{\sqrt{n}}{\ln n} \right\}_{n=1}^{\infty}$

83. $\left\{ \ln\left(\dfrac{3n}{n+1} \right) \right\}_{n=1}^{\infty}$

84. $\left\{ \dfrac{n^2 \sin n}{n^5 + 7} \right\}_{n=1}^{\infty}$

85. $\left\{ \dfrac{1}{3n + 7} \right\}_{n=1}^{\infty}$

86. $\left\{ \ln(n^2 - 7) - \ln(3n^2 + n + 9) \right\}_{n=1}^{\infty}$

87. $\left\{ \arctan(n^2 + 1) \right\}_{n=1}^{\infty}$

5

88. $\left\{e^{-2n}\right\}_{n=1}^{\infty}$

89. $\left\{\dfrac{4}{\ln n}\right\}_{n=1}^{\infty}$

90. $\left\{\dfrac{\ln n}{n}\right\}_{n=1}^{\infty}$

91. $\left\{\dfrac{\sqrt{n}}{(\ln n)^2}\right\}_{n=1}^{\infty}$

92. $\left\{(e^n + n)^{\frac{1}{n}}\right\}_{n=1}^{\infty}$

93. $\left\{n^{\frac{1}{n}}\right\}_{n=1}^{\infty}$

94. $\left\{\dfrac{\sin^2 n}{n^2 + 3}\right\}_{n=1}^{\infty}$

95. $\left\{n \cos\left(\dfrac{1}{n}\right)\right\}_{n=1}^{\infty}$

Series: Find the **sum** for each of the following series (all of which converge):

96. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 3^n}{6^n}$

97. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{4^n} - \frac{1}{7^n}$

98. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^{n-1}}{3^{n+1}}$

99. $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+2}}{2^{4n-1}}$

100. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$

101. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3 \cdot 2^n}$

102. $\displaystyle\sum_{n=1}^{\infty} e^{\frac{1}{n}} - e^{\frac{1}{n+1}}$

103. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{3^{2n-1}}$

104. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$

105. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{9^{n-1}}$

106. $\displaystyle\sum_{n=1}^{\infty} 2^{-2n}$

More Series: Determine whether each of the following series **converge** or **diverge**. Name any convergence test(s) you use, and justify that it's legal to use them:

107. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{2^n}$

108. $\displaystyle\sum_{n=1}^{\infty} \frac{2n + \ln n}{n + 2010}$

109. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

110. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)^2 - n}$

111. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2 + 1}$

112. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n^2}{n!}$

113. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

114. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 1}{2n^2 \sqrt{n} + 9}$

115. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n} + 3}{4n^2 - 2}$

116. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{19} + 40n^6 + 4n^3 + 19}{4 + 17n^5 + n^{20}}$

117. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n(\sqrt{n} + 1)}$

118. $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{2^n n!}$

119. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(\ln 2)^n}$

120. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$

121. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{e^n}$

122. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n \ln n}$

123. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{3n + 2}$

124. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$

125. $\displaystyle\sum_{n=1}^{\infty} n e^{-n^2}$

126. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{\ln n}$

127. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^{4n}}$

128. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{7}{8}}}$

129. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$

130. $\displaystyle\sum_{n=1}^{\infty} e^{-2n}$

131. $\displaystyle\sum_{n=1}^{\infty} \frac{1 + 3n^3}{n^5}$

132. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(\ln n)^2}$

133. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$

134. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^7}$

135. $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{1 + n^2}$

136. $\displaystyle\sum_{n=1}^{\infty} \frac{2 + \sin n}{n^2}$

137. $\displaystyle\sum_{n=1}^{\infty} \frac{n^7}{e^n}$

138. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{3^n}$

139. $\displaystyle\sum_{n=1}^{\infty} \frac{2n + 5}{5n^3 + 3n^2}$

140. $\displaystyle\sum_{n=1}^{\infty} (e^n + n)^{\frac{1}{n}}$

141. $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

142. $\displaystyle\sum_{n=1}^{\infty} n^{\frac{1}{n}}$

143. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

144. $\displaystyle\sum_{n=1}^{\infty} \frac{n \sin^2 n}{n^2 + 3}$

145. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

146. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n^2}$

147. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n + 7}$

148. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{2^n + 3^n}$

149. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln n}}$

150. $\displaystyle\sum_{n=1}^{\infty} \frac{5 \cdot 2^n + 6^n}{n2^n}$

151. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 - 1}{3n^2 + 1}$

152. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{25 + n^2}$

153. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

154. $\displaystyle\sum_{n=1}^{\infty} \frac{(3n)! + 4^{n+1}}{(3n+1)!}$

155. $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$

156. $\displaystyle\sum_{n=1}^{\infty} \pi^{-n} e^n$

157. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(2n-1)!}$

158. $\displaystyle\sum_{n=1}^{\infty} 3 + \frac{1}{3^n}$

159. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

160. $\displaystyle\sum_{n=1}^{\infty} e^{\frac{1}{n}}$

161. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

162. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^3 7^n}$

163. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n}{(n+1)^2}$

164. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n^2}{(n+1)!}$

165. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n!}$

166. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{5^n}$

167. $\displaystyle\sum_{n=2}^{\infty} \left(-\frac{3}{4}\right)^n$

168. $\displaystyle\sum_{n=1}^{\infty} \cos(\pi n)$

169. $\displaystyle\sum_{n=2}^{\infty} e^{\left(\frac{\sin n}{n}\right)}$

170. $\displaystyle\sum_{n=2}^{\infty} \frac{9^n}{(-2)^{n+1}n}$

171. $\displaystyle\sum_{n=2}^{\infty} \frac{3 \cdot 7^n - n^6}{n^7 7^n}$

172. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)^n n!}{(2n)!}$

173. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n (n!)^3}{(2n)! n^n}$

Even More Series: Determine whether each of the following series **converges absolutely**, **converges conditionally**, or **diverges**. Justify your answers.

174. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{5n}$

175. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2^n}$

176. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{7n - 3}$

177. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{n \ln n}$

178. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\ln(n + 1)}$

179. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n}{n + 3^n}$

180. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{10n+1}$

181. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$

182. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(\pi n)}{n}$

183. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\ln n}{n}$

184. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{\cosh n}$

185. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{2^{n^2}}$

186. $\displaystyle\sum_{n=2}^{\infty}\frac{n(-3)^{2n+1}}{10^n}$

187. $\displaystyle\sum_{n=2}^{\infty}2^{\ln n}\left(\frac{1}{2}\right)^n$

188. $\displaystyle\sum_{n=1}^{\infty}\frac{7^n}{n^n}$

189. $\displaystyle\sum_{n=1}^{\infty}\frac{(-2)^n}{n+3^n}$

190. $\displaystyle\sum_{n=1}^{\infty}\frac{e^{2n}}{n^n}$

191. $\displaystyle\sum_{n=1}^{\infty}\frac{(-4)^{2n+1}}{n10^n}$

192. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\ln(n^2)}{n^3}$

193. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\arctan n}{n+2^n}$

194. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n(n+1)}$

195. $\displaystyle\sum_{n=1}^{\infty}\frac{(n+2)!}{3^n(n!)^2}$