Name:_____

Amherst College

DEPARTMENT OF MATHEMATICS

Math 12

Midterm Exam #1

February 23, 2011

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		30
2		30
3		40
Total		100

1. [30 Points] Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x \to 0} \dfrac{xe^x - x}{x \sin(3x)}$

(b) $\lim_{x \to \infty} \left(1 - \dfrac{3}{x}\right)^x$

1. (Continued) Evaluate the following **limit**. Please justify your answer. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(c) $\lim_{x\to\infty} (e^x + x)^{\frac{1}{x}}$

2. [30 Points] Compute each of the following **definite integrals**. Please simplify your answer.

(a) $\displaystyle\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\, dx$

(b) $\displaystyle\int_0^{\ln 7} \tanh x\, dx$

2. (Continued) Compute the following **definite integral**. Please simplify your answer.

(c) $\int_1^e (\ln x)^2 \, dx$

3. [40 Points] Compute each of the following **indefinite integrals**.

(a) $\displaystyle\int \left(x + \frac{1}{e^{3x}}\right)^2 \, dx$

(b) $\displaystyle\int x \, \arctan x \, dx$

3. (Continued) Compute the following **indefinite integral**.

(c) $\displaystyle\int \frac{1}{(x^2+4)^{\frac{5}{2}}}\,dx$

3. (Continued) Compute the following **indefinite integral**.

(d) $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

****

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

****

OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\int \frac{1}{\sqrt{e^x - 1}}\, dx$

OPTIONAL BONUS #2 Compute the following **indefinite integral**.

2. $\int \frac{xe^x}{\sqrt{1 + e^x}} dx$