Name:_____

Amherst College

DEPARTMENT OF MATHEMATICS

Math 12

Midterm Exam #1

February 24, 2010

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		6
2		24
3		25
4		45
Total		100

1. [6 Points] Compute the **derivative** for the following function. Do not simplify your answer.

$$f(x) = \arctan(4x) \cdot \arcsin(3x)$$

2. [24 Points] Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x\to\infty} \dfrac{\arctan x}{\dfrac{1}{x}+1}$

(b) $\lim_{x\to 0} \dfrac{\sin(3x)}{9\cos x - 5x - 9}$

2. (Continued) Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(c) $\lim_{x\to\infty} x\left(2e^{\frac{1}{x}} - 2\right)$

(d) $\lim_{x\to 0} (1+3x)^{\frac{2}{x}}$

3. [25 Points] Compute each of the following **definite integrals**. Please simplify your answer.

(a) $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\, dx$

(b) $\displaystyle\int_1^e \ln x\, dx$

(c) $\displaystyle\int_0^{\ln 3} \sinh x\, dx$

4. [45 Points] Compute each of the following **indefinite integrals**.

(a) $\int \frac{e^x}{1+e^{2x}}\, dx$ [Hint: $e^{2x} = (e^x)^2$]

(b) $\int x^2 e^x\, dx$

4. (Continued) Compute each of the following **indefinite integrals**.

(c) $\displaystyle\int \frac{1}{(4-x^2)^{\frac{3}{2}}}\,dx$

(d) $\displaystyle\int \sin^3 x\ \cos^4 x\ dx$

4. (Continued) Compute the following **indefinite integral**.

(e) $\displaystyle\int x \ \arcsin x \ dx$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\int \frac{x^3}{1-\sin(x^2)}\,dx$

OPTIONAL BONUS #2 Compute the following **indefinite integral**.

2. $\int \frac{1}{x^{\frac{3}{2}}\left(1+x^{\frac{1}{3}}\right)}dx$