

Name: _____

Amherst College
DEPARTMENT OF MATHEMATICS
Math 12 Final Exam
May 13, 2010

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

- You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, or $e^{3\ln 3}$ should be simplified.

- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		10
2		15
3		20
4		10
5		20
6		10
7		5
8		10
9		15
10		15
11		15
12		10
Total		155

1. [10 Points] Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x \rightarrow 3} \frac{\arctan(x - 3)}{x^2 - 9}$

(b) $\lim_{x \rightarrow \infty} (e^x + 1)^{\frac{1}{x}}$

2. [15 Points] Evaluate each of the following **integrals**.

(a) $\int_0^{\ln 7} \frac{\sinh x}{\cosh x} dx$

(b) $\int x \arcsin x dx$

3. [20 Points] For each of the following **improper integrals**, determine whether it converges or diverges. If it converges, find its value.

(a) $\int_7^{\infty} \frac{1}{x^2 - 8x + 19} dx$

(b) $\int_0^4 \frac{3x + 2}{x^2 - x - 12} dx$

4. [10 Points] Find the **sum** of each of the following series (which do converge):

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{9^{n-1}}$$

$$(b) \sum_{n=0}^{\infty} \frac{3^n}{n!}$$

$$(c) \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n} (2n+1)!}$$

5. [20 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Justify your answers.

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$$

$$(b) \sum_{n=1}^{\infty} \frac{e^n + 9}{n+1}$$

5. (Continued) In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Justify your answers.

$$(c) \sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n}$$

$$(d) \sum_{n=1}^{\infty} (-1)^n \frac{n^3 + 8}{n^7 - 9}$$

6. [10 Points] Find the **Interval** and **Radius** of Convergence for the following power series. Analyze carefully and with full justification.

$$\sum_{n=0}^{\infty} \frac{(-1)^n (3x - 1)^n}{n^2 \cdot 2^n}$$

7. [5 Points] Consider the function $f(x)$ that satisfies the following

$$f(3) = 2$$

$$f'(3) = -3$$

$$f''(3) = \frac{6}{7}$$

$$f'''(3) = -1$$

Find the **Taylor polynomial of degree 3** for $f(x)$ centered at $a = 3$.

8. [10 Points] **MacLaurin Series:** Please analyze with detail and justify carefully.

(a) Find the **MacLaurin series** representation for $f(x) = x \ln(1 + x)$.

Your answer should be in sigma notation $\sum_{n=0}^{\infty}$.

(b) Use the MacLaurin Power Series representation for $f(x) = x \ln(1 + x)$ from Part(a) to

Estimate $\int_0^1 x \ln(1 + x) dx$ with error less than $\frac{1}{10}$.

9. [15 Points] Volumes of Revolution

(a) Consider the region bounded by $y = e^x$, $y = \ln x$, $x = 1$, and $x = 2$. Rotate the region about the y -axis. **Compute** the **volume** of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.

(b) Consider the same region bounded by $y = e^x$, $y = \ln x$, $x = 1$, and $x = 2$. Rotate the region about the vertical line $x = -3$. Set up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Cylindrical Shells Method.

10. [15 Points] Consider the Parametric Curve represented by $x = e^t - t$ and $y = 4e^{t/2}$.

(a) Find the **arclength** of this parametric curve for $0 \leq t \leq 1$.

(b) Set up, **BUT DO NOT EVALUATE!!**, the definite integral representing the **surface area** of the solid obtained by rotating this curve about the x -**axis**, for $0 \leq t \leq 1$.

11. [15 Points] For each part, **sketch** the Polar curve(s), and answer the related questions:

(a) **Compute** the **area** enclosed by the cardioid $r = 1 + \sin \theta$.

(b) Set up, **BUT DO NOT EVALUATE!!**, the definite integral representing the **area** bounded inside $r = 2 + 2 \cos \theta$ and outside $r = 3$.

12. [10 Points] Find the general solution for each of the following **differential equations**.

(a) $\frac{dy}{dx} = (1 + y^2)e^x$

(b) $\frac{dy}{dx} + 2xy = e^{-x^2}$