Math 121-Benedetto

What you need to know for Exam 3

You should know Sections 11.8–11.10, Power Series and their Applications. This exam will not explicitly cover the material from earlier sections, but of course it will still be assumed that you know how to deal with convergence of infinite series, exponentials, logarithms, trig and inverse trig functions, L'Hôpital's rule, substitution, and so on. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- 11.8: Power Series. Know the definition, and remember that it includes not just series of the form $\sum c_n x^n$ but also series like $\sum c_n (x-a)^n$, where a is a constant, called the **center** of the power series. Know how to find the Radius and Interval of convergence of a given power series. Remind yourself what the 3 options are for the (domain) intervals of convergence. Check how to analyze convergence at endpoints for finite case.

- 11.9: Power Series Representations of Functions. Know how to use power series you already know (like that for $1/(1-x)$, or (in later sections) e^x, $\cos x$, $\sin x$) to find power series for other functions by the following five operations: (1) substituting monomials like $-3x$ or $2x^3$ for x, or (2) multiplying by a polynomial, or (3) adding two power series (centered at the same center), or (4) integrating (usually followed by evaluating at $x = a$ to find the constant C from integration), or (5) differentiating. Know how each of these operations may affect the Radius of Convergence. See the Theorem.

- 11.10: Taylor and MacLaurin Series. Know the following:

  -Computing Taylor/MacLaurin Series using the Definition (Chart Method)

  -Our 6 memorized MacLaurin Series

  -Applications, including new series from known ones, new derivatives, new substitutions, new (indefinite) integrals, new sums (beyond geometric), and estimates for values or (definite) integrals using Alternating Series Estimation Theorem, and finally Limits using Series.

NOTE: If you're asked for the **sum** of a series, on Exam 2 that meant it would be a geometric series. Now we have another possibility: that you can get that series by plugging in x equals some specific number in some specific power series. For example, $\displaystyle\sum_{n=0}^{\infty} \frac{5^{n-2}}{2^{2n} \cdot n!}$ can be rewritten as $\displaystyle\frac{1}{25}\sum_{n=0}^{\infty} \frac{\left(\frac{5}{4}\right)^n}{n!} = \frac{1}{25}e^{\frac{5}{4}}$. Similarly, $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 7^n}{n!} = \sum_{n=0}^{\infty} \frac{(-7)^n}{n!} = e^{-7}$.

RECALL: $\displaystyle\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = e$ and $\displaystyle\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \frac{1}{e}$

Common Types of Problems to Prepare, Know how to . . .

- How to determine the three possibilities for Interval and Radius of Convergence.
  - *Collapsed* Interval, $I = \{\text{center point } a\}$ with $R = 0$
  - *Finite* Interval centered at the center point a with Radius R
  - *Infinite* Interval $I = (-\infty, \infty)$ with $R = \infty$
- How to decide between Geometric Series or Ratio Test to determine the Domain.
- How to manually check convergence at the endpoints for finite interval domain case. Understand why the Ratio Test is inconclusive at the endpoints for finite interval.
- How to write the three different conclusive statements to justify each of the three Interval of Convergence cases for the Ratio Test conclusions. Highlighted in HW 15.
- How to create a Power Series example which satisfies a given requirement. Ex: *Create a Power Series centered at* $a = 5$ *with Radius of Convergence equal to* 0 or *Create a Power Series centered at* $a = 7$ *with Infinite Radius of Convergence.* Think: size!
- How to Differentiate/Integrate Power Series term-by-term. Know how the Radius of Convergence is affected.
- How to write the series in Long expanded form, plug in center point, to compute $+C$ in a series derivation using integration.
- How to use 4 different methods to find a function's Power Series Representation: that is
  - Substitution into a known MacLaurin Series
  - Differentiating another Series
  - Integrating another Series
  - Using the MacLaurin/Taylor Series Definition/*Chart Method*. Write the formula.
- How to find the Radius of Convergence for newly derived Series. There may be a shortcut to avoid using the (longer) Ratio Test, by using substitution into known series.
- How to memorize the six known MacLaurin Series. Know I and R for each.
- How to use multiple methods to derive the Series for $\sin x$ and $\cos x$ **three** different ways, $\ln(1 + x)$ **two** different ways, and others like $\arctan x$ or e^x one way.
- How to use Series for **Applications:**
  - How to compute the **Sum** Series value for convergent series (which are not geometric) based on algebra manipulations and/or pattern matching to the known six MacLaurin Series.
  - How to **Estimate** Values or Definite Integrals using the **Alternating Series Estimation Theorem**. Make clear which is full series or estimate using a few terms.
  - How to use Series to compute **Limits** of complicated function combinations, and how to check your answer using L'Hôpital's Rule.