Math 121, D. Benedetto

What you need to know for Exam 1

You should know Section 4.5, Sections 6.1–6.8, except Section 6.5/6.7. Also know Sections 7.1-7.3. The test will not explicitly cover material from earlier sections (or from Appendix D), but of course it will be assumed that you know what limits, derivatives, and integrals are, that you know how to use differentiation rules, and that you know some trigonometry. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- 4.5: The Substitution Rule, for both indefinite and definite integrals.
- 6.1: Know what one-to-one means and what an inverse function is. Know how the domain and range of f relate to the domain and range of f^{-1}.
- 6.2: Know the algebraic properties and limit properties of exponentials. The graph, derivative, and antiderivative of e^x. The chain rule with $e^{u(x)}$.
- 6.3: Algebraic properties of logarithms. The notion of $\ln(x)$ as the inverse function of e^x. The fact that $\ln e = 1$ and $\ln 1 = 0$. The graph and the limit properties of $\ln(x)$.
- 6.4: Derivative of $\ln(x)$ and antiderivative of $1/x$. The chain rule with $\ln(u(x))$. Know that $a^b = e^{b \ln a}$. See the Review Handout on e^x and $\ln x$.
- 6.6: The definition of inverse sine and tangent (not cosine or secant). The limit properties and the derivatives of $\arcsin x$ and $\arctan x$. Know the corresponding antiderivative formulas.
- 6.7: SKIPPED Hyperbolic Functions
- 6.8: L'Hôpital's Rule. Know the key hypothesis (that the original limit is one of the indeteriminate forms $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$). Know how to apply it. Know how to use it to attack other indeterminate limits, like $0\cdot\infty$, 0^0, ∞^0, and 1^∞. Know NOT to apply it to NON-indeterminate forms (like $\frac{\infty}{0}$, $\infty \cdot \infty$, ∞^∞, 0^∞, and so on).
- Appendix D: This is just standard trig, so we didn't cover it in class, but you of course need to know all the fundamentals here. Know your trig values at standard angles.
- 7.1: Know the Integration by Parts rule, both for indefinite and for definite integrals. The LIPET mnemonic may help in choosing your u. Be able to apply Parts two or more times in a row.
- 7.2: Trig Integrals. Know how to integrate products of sines and cosines, like $\sin^5 x \cos^4 x$. See the book for a summary. Also know how to integrate simpler products of tangents and secants. Know the relevant trig identities for each case. What are the half-angle identities?
- 7.3: Trig substitution. Know what substitution to make given the forms shown in the box in the book. Triangle arguments (and trig identities, like $\sin(2x) = 2\sin x \cos x$, if needed) to convert the final answer back to the original variable. Really practice here. Most of these integrals reduce at some point to integrals from previous sections, especially Trig Integrals, products of powers of sines and cosines. Do not need Complete the Square Algebra (for now).

Common Types of Problems to Prepare, Know how to ...

- How to Prove Derivative Facts for Inverse Functions with $\ln x, \arcsin x, \arctan x$ using a "L.I.D.S." style proof. Ex: **Prove** $\dfrac{d}{dx}\arcsin(5x) = \dfrac{5}{\sqrt{1-25x^2}}$.
- How to Prove the a-Rules for Integration, using algebra and u-sub

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+C \quad \text{and} \quad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)+C$$

- How to use L'Hôpital's Rule for Indeterminate stacks $\dfrac{0}{0}$ and $\pm\dfrac{\infty}{\infty}$ (including *Repeated Use* and complex Chain Rule pieces), as well as Indeterminate Products $0\cdot\infty$ and Indeterminate Powers $1^\infty, 0^0$ and ∞^0. Study how one type reduces to another ...
- How to use Integration by Parts to integrate products of different "LIPET" combos, including especially $\ln x$, $\arctan x$ and $\arcsin x$.
- How to compute Trigonometric Integrals using both ODD and EVEN Power techniques for integrals of the form $\displaystyle\int \sin^m x \cos^n x\ dx$ where integers $m, n \geq 0$. Which Identities, where?
- How to compute Trigonometric Substitutions using either sine or tangent substitutions. Check the Relevant Identities. When is Trig Sub used?
- How to compute Integral with several *Layered* Techniques. For instance, $\displaystyle\int x\arcsin x\ dx$ starts as an I.B.P and then becomes (sine) Trig Sub, and then ultimately finishes as an Even powered Integral. Study which techniques apply to which integral types.
- How to evaluate exponentials, logs, and Inverse Trig functions at any standard input value.

Tips

- Do not drop limit signs. Mark L'H application clearly each time.
- Study the Integration Reference Sheet carefully; know all of your basic integrals. No hesitation allowed! Know your trig. identities. Remember $+C$.
- Precisely learn each integration technique. Using these techniques, practice turning complicated integrals into the more basic ones on the Reference Sheet.
- Practice enough in order to *immediately* recognize which technique of integration is needed. There may be some flexibility here.
- For definite integrals, make sure that you change your limits of integration **every** time you make a substitution. They could be layered with multi-substitutions. in a single problem, so be careful. Otherwise, mark limits correctly. No mixing of variables.
- Know when ... and when **not** to apply L'H Rule.
- For L'Hôpital's rule, the main thing is to check that the limit in question looks like $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$. So don't forget to check that. Then, be careful when differentiating. (Differentiate numerator and denominator **separately**; don't apply the quotient rule to the whole thing.)