

Homework #4**Due Friday, February 7th** in Gradescope by 11:59 pm ET**Goal:** Reviewing Inverse Trigonometric Functions and Limits (no L'Hopital's Rule yet)**FIRST:** Read through and understand the following Examples.

Ex:

$$\begin{aligned}
\int_e^{e^3} \frac{1}{x(3 + (\ln x)^2)} dx &= \int_1^3 \frac{1}{3 + u^2} du \stackrel{\text{a-rule}}{=} \frac{1}{\sqrt{3}} \arctan\left(\frac{u}{\sqrt{3}}\right) \Big|_1^3 \\
&= \frac{1}{\sqrt{3}} \left(\arctan\left(\frac{3}{\sqrt{3}}\right) - \arctan\left(\frac{1}{\sqrt{3}}\right) \right) \\
&= \frac{1}{\sqrt{3}} \left(\frac{\pi}{3} - \frac{\pi}{6} \right) = \frac{1}{\sqrt{3}} \left(\frac{2\pi}{6} - \frac{\pi}{6} \right) = \boxed{\frac{\pi}{6\sqrt{3}}}
\end{aligned}$$

$$\begin{aligned}
u &= \ln x \\
du &= \frac{1}{x} dx
\end{aligned}$$

$$\begin{aligned}
x = e &\Rightarrow u = \ln e = 1 \\
x = e^3 &\Rightarrow u = \ln e^3 = 3
\end{aligned}$$

Ex:

$$\int \frac{e^{3x}}{4 + e^{3x}} dx = \frac{1}{3} \int \frac{1}{u} du = \frac{1}{3} \ln |u| + C = \boxed{\frac{1}{3} \ln |4 + e^{3x}| + C}$$

$$\begin{aligned}
u &= 4 + e^{3x} \\
du &= 3e^{3x} dx \\
\frac{1}{3} du &= e^{3x} dx
\end{aligned}$$

Ex:

$$\int \frac{e^{3x}}{4 + e^{6x}} dx = \int \frac{e^{3x}}{4 + (e^{3x})^2} dx \quad \text{look for the perfect square to ready for arctan rule}$$

$$\frac{1}{3} \int \frac{1}{4 + u^2} du \stackrel{\text{a-rule}}{=} \frac{1}{3} \left(\frac{1}{2} \arctan\left(\frac{u}{2}\right) \right) + C = \boxed{\frac{1}{6} \arctan\left(\frac{e^{3x}}{2}\right) + C}$$

$$\begin{aligned}
u &= e^{3x} \\
du &= 3e^{3x} dx \\
\frac{1}{3} du &= e^{3x} dx
\end{aligned}$$

Next, Compute each of the following Integrals. Simplify.

1. $\int_2^{2\sqrt{3}} \frac{1}{\sqrt{16-x^2}} dx$
2. $\int_0^{\ln 3} \frac{e^x}{3+e^{2x}} dx$
3. $\int_0^{\ln \sqrt{3}} \frac{e^x}{\sqrt{4-e^{2x}}} dx$
4. $\int_4^{4\sqrt{3}} \frac{1}{16+x^2} dx$
5. $\int \frac{x}{\sqrt{1-x^4}} dx$
6. $\int \frac{x^2}{x^2+4} dx$
7. $\int \frac{2x^2+5}{x^2+1} dx$
8. $\int \frac{1}{(1+x^2)(5+(\arctan x)^2)} dx$
9. $\int_3^9 \frac{1}{\sqrt{x}(x+9)} dx$
10. $\int \frac{x^2+x+1}{x^2+4} dx$

Compute each of the following Limits. Simplify. Use arrows to justify the size arguments.

11. $\lim_{x \rightarrow 5^+} \frac{1}{x-5}$
12. $\lim_{x \rightarrow 5^-} \frac{1}{x-5}$
13. $\lim_{x \rightarrow 8^+} \ln |x-8|$
14. $\lim_{x \rightarrow 8^-} \ln |x-8|$
15. $\lim_{x \rightarrow 3^+} e^{\frac{2}{x-3}}$
16. $\lim_{x \rightarrow 3^-} e^{\frac{2}{x-3}}$
17. $\lim_{x \rightarrow \infty} \ln \left(1 - \arctan \left(\frac{5}{x^4} \right) \right)$
18. $\lim_{x \rightarrow \infty} \ln \left(\frac{\pi}{2} - \arctan x \right)$
19. $\lim_{x \rightarrow 4^-} \ln |\ln |x-4||$
20. $\lim_{x \rightarrow 0^+} \arctan \left(\frac{\ln x}{5} \right)$

21. Present two different methods to Prove that $\int \frac{1}{4+x^2} dx = \frac{1}{2} \arctan \left(\frac{x}{2} \right) + C$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Jude, SMUDD 207

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Aaron, SMUDD 207

Wednesday: 1:00–3:00 pm

6–7:30 pm TA Jude, SMUDD 207

7:30–9:00 pm TA Aaron, SMUDD 207

Thursday: none for Professor

extras may be added, TBD weekly

7:30–9:00 pm TA DJ, SMUDD 207

Friday: 12:00–3:00 pm

7:30–9:00 pm TA DJ, SMUDD 207

- Please stop by for help! Try to attend at least one office hour for me and at least one for the Math Fellows each week.
- You can also find help at the Math Fellow (Aaron, Jude DJ) sessions.