

Homework #3

Due **Wednesday, February 5th** in Gradescope by 11:59 pm ET

Goal: Solidifying Calculus for Inverse Sine and Inverse Tangent.

FIRST: Read through and understand the following two Derivative proofs.

Ex: **PROVE** that $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$

Proof: Let $y = \arctan x$ Looking to solve for $\frac{dy}{dx}$

Invert $\tan y = x$

Differentiate $\frac{d}{dx} (\tan y) = \frac{d}{dx} (x)$

$$\sec^2 y \cdot \frac{dy}{dx} = 1$$

Solve $\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + (\tan y)^2} = \frac{1}{1 + x^2}$

Ex: **PROVE** that $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$

Proof: Let $y = \arcsin x$ Looking to solve for $\frac{dy}{dx}$

Invert $\sin y = x$

Differentiate $\frac{d}{dx} (\sin y) = \frac{d}{dx} (x)$

$$\cos y \cdot \frac{dy}{dx} = 1$$

Solve $\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1 - \sin^2 y}} = \frac{1}{\sqrt{1 - (\sin y)^2}} = \frac{1}{\sqrt{1 - x^2}}$

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Differentiate the following functions for 1-4. Simplify.

1. $f(x) = \tan^{-1}(x^2)$ 2. $f(x) = (\tan^{-1}(x))^2$

3. $y = x \sin^{-1} x + \sqrt{1-x^2}$ 4. $f(x) = \ln \left(1 - \arcsin \left(\frac{2}{x^4} \right) \right)$

5. Find the value of the expression $\tan \left(\sin^{-1} \left(\frac{2}{3} \right) \right)$

6. Simplify the expression $\sin(\tan^{-1} x)$

7. Compute the Second Derivative for $f(x) = \arctan(2x)$

8. Compute the Second Derivative for $f(x) = \arcsin(6x)$

9. **Prove** that $\frac{d}{dx} \sin^{-1}(3x) = \frac{3}{\sqrt{1-9x^2}}$

10. **Prove** that $\frac{d}{dx} \tan^{-1}(5x) = \frac{5}{1+25x^2}$

11. Use Integration to **Justify** that $\int \frac{1}{3+x^2} dx = \frac{1}{\sqrt{3}} \arctan \left(\frac{x}{\sqrt{3}} \right) + C$

Compute each of the following Integrals. Simplify.

12. $\int \frac{x^2}{x^2+1} dx$

13. $\int \frac{x+1}{x^2+1} dx$

14. $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{8}{1+x^2} dx$

15. $\int_0^{\frac{1}{2}} \frac{\arcsin x}{\sqrt{1-x^2}} dx$

16. $\int \frac{1}{\sqrt{1-x^2} \cdot \sin^{-1} x} dx$

17. $\int_1^3 \frac{1}{\sqrt{x}(1+x)} dx$

18. $\int_0^{\ln 3} \frac{e^x}{1+e^x} dx$

19. $\int_0^{\frac{1}{2} \ln 3} \frac{e^x}{1+e^{2x}} dx$

20. $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}} dx$

21. $\int_3^{3\sqrt{3}} \frac{1}{\sqrt{36-x^2}} + \frac{1}{9+x^2} dx$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Jude, SMUDD 207

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Aaron, SMUDD 207

Wednesday: 1:00–3:00 pm

6–7:30 pm TA Jude, SMUDD 207

7:30–9:00 pm TA Aaron, SMUDD 207

Thursday: none for Professor

extras may be added, TBD weekly

7:30–9:00 pm TA DJ, SMUDD 207

Friday: 12:00–3:00 pm

7:30–9:00 pm TA DJ, SMUDD 207

- Please do not wait until the last night to start.
- Please stop by for help! Please try the homework before you come by though. Final Answer keys are posted on the webpage. Please do **not** look at them unless you have completed the problems. **They are not a replacement for my help or your understanding.**
- You can also find help at the Math Fellow sessions or the QCenter hours with Tim St. Onge