Math 121, Section(s) 01, Spring 2025

Homework #18

Due **Friday, April 18th** in Gradescope by 11:59 pm ET

Goal: Exploring More complicated Sums and Limits using Series.

FIRST: Read through and understand the following Examples.

Find the **sum** of each of the following series (which do converge). Simplify.

TIP: For these values, it could help to write out the general MacLaurin Series in x first.

Ex: $-1+\frac{1}{3}-\frac{1}{5}+\frac{1}{7}-\frac{1}{9}+\ldots=-\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots\right)=-\arctan 1=\boxed{-\frac{\pi}{4}}$

Ex: $\sum_{n=0}^{\infty}\frac{(-1)^n(\ln 64)^n}{2^{n+1}\,n!}=\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n(\ln 64)^n}{2^n\,n!}=\frac{1}{2}\sum_{n=0}^{\infty}\frac{\left(-\frac{\ln 64}{2}\right)^n}{n!}=\frac{1}{2}e^{-\frac{\ln 64}{2}}=\frac{1}{2}e^{-\frac{1}{2}\ln(64)}$

$=\frac{1}{2}\cancel{e}^{\cancel{\ln}\left((64)^{-\frac{1}{2}}\right)}=\frac{1}{2}\cdot(64)^{-\frac{1}{2}}=\frac{1}{2}\cdot\frac{1}{(64)^{\frac{1}{2}}}=\frac{1}{2}\cdot\frac{1}{\sqrt{64}}=\frac{1}{2}\cdot\frac{1}{8}=\boxed{\frac{1}{16}}$

Ex: $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}\,\pi^{2n}}{9^n\,(2n+1)!}=-\sum_{n=0}^{\infty}\frac{(-1)^n\,\pi^{2n}}{3^{2n}\,(2n+1)!}=-\sum_{n=0}^{\infty}\frac{(-1)^n\left(\frac{\pi}{3}\right)^{2n}}{(2n+1)!}\cdot\frac{\frac{\pi}{3}}{\frac{\pi}{3}}$

$=-\frac{1}{\frac{\pi}{3}}\sum_{n=0}^{\infty}\frac{(-1)^n\left(\frac{\pi}{3}\right)^{2n+1}}{(2n+1)!}=-\frac{3}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\left(\frac{\pi}{3}\right)^{2n+1}}{(2n+1)!}=-\frac{3}{\pi}\sin\left(\frac{\pi}{3}\right)^{\nearrow\frac{\sqrt{3}}{2}}=\boxed{-\frac{3\sqrt{3}}{2\pi}}$

Ex: $1+1-\frac{\pi^2}{2!}+\frac{\pi^4}{4!}-\frac{\pi^6}{6!}+\frac{\pi^8}{8!}-\ldots=\overset{\text{extra}}{1}+\cos\pi=1+\cos\pi^{\nearrow -1}=1-1=\boxed{0}$

Ex: $-\frac{2}{2}+\frac{2}{3}-\frac{2}{4}+\frac{2}{5}-\ldots=2\left(-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\ldots\right)=2\left(\ln(1+1)-1\right)=\boxed{2\ln 2-2}$

Ex: Use series to compute this **Limit**.

$$\lim_{x\to 0}\frac{x^2+x-\arctan x}{1-3x-e^{-3x}}=\lim_{x\to 0}\frac{x^2+\cancel{x}-\left(\cancel{x}-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\right)}{\cancel{1}-\cancel{3x}-\left(\cancel{1}-\cancel{3x}+\frac{3^2x^2}{2!}-\frac{3^3x^3}{3!}+\ldots\right)}$$

$$=\lim_{x\to 0}\frac{x^2+\frac{x^3}{3}-\frac{x^5}{5}+\ldots}{-\frac{3^2x^2}{2!}+\frac{3^3x^3}{3!}-\frac{3^4x^4}{4!}+\ldots}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}=\lim_{x\to 0}\frac{1+\frac{x}{3}-\frac{x^3}{5}+\ldots\nearrow 0}{-\frac{9}{2!}+\frac{3^3x}{3!}-\frac{3^4x^2}{4!}+\ldots\nearrow 0}=\frac{1}{-\frac{9}{2}}=\boxed{-\frac{2}{9}}$$

Continue on and Complete the next page of problems

Find the **sum** of each of the following series (which do converge). Simplify.

1. $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots$

2. $\sum_{n=0}^{\infty}\frac{(-1)^n\ \pi^{2n+1}}{9^n\ (2n)!}$

3. $-\frac{\pi^3}{3!}+\frac{\pi^5}{5!}-\frac{\pi^7}{7!}+\frac{\pi^9}{9!}-\ldots$

4. $-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots$

5. $-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\frac{1}{6}-\ldots$

6. $\sum_{n=0}^{\infty}\frac{(-1)^n\,(\ln 8)^n}{3^{n+1}\ n!}$

7. $\sum_{n=0}^{\infty}\frac{(-1)^n\ \pi^{2n}}{(36)^n\ (2n+1)!}$

8. $\frac{1}{6}-\frac{1}{2(6)^2}+\frac{1}{3(6)^3}-\frac{1}{4(6)^4}+\ldots$

9. $1-e+\frac{e^2}{2!}-\frac{e^3}{3!}+\frac{e^4}{4!}-\frac{e^5}{5!}+\ldots$

10. $-\frac{\pi^2}{2!}+\frac{\pi^4}{4!}-\frac{\pi^6}{6!}+\frac{\pi^8}{8!}-\ldots$

11. $\sum_{n=0}^{\infty}\frac{(-1)^n\ \pi^{2n}}{(2n)!}$

12. $\sum_{n=0}^{\infty}\frac{1}{e^n}$

13. $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}\ 2^{n+1}\ (\ln 9)^n}{n!}$

14. $4-\frac{4}{3}+\frac{4}{5}-\frac{4}{7}+\frac{4}{9}-\ldots$

15. $\sum_{n=0}^{\infty}\frac{e^6\ (x-6)^n}{n!}$ (answer will be in x)

16. $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}\ \pi^{2n+1}}{9\ (2n)!}$

17. $\sum_{n=0}^{\infty}\frac{1}{3!\ \pi^n}$

18. $-\pi+\frac{\pi^3}{3!}-\frac{\pi^5}{5!}+\ldots$

19. $1+1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$

20. $2-1+\frac{2}{3}-\frac{2}{4}+\frac{2}{5}-\ldots$

21. $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\ldots$

22. $\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n+1}}{\left(\sqrt{2}\,\right)^{4n}\ (2n)!}$

23. Use Series to Compute $\lim_{x\to 0}\frac{xe^x-\arctan x}{\ln(1+5x)-5x}$ Check your answer using L'H Rule

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Jude, SMUDD 207

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Aaron, SMUDD 207

Wednesday: 1:00-3:00 pm

6–7:30 pm TA Jude, SMUDD 207

7:30–9:00 pm TA Aaron, SMUDD 207

Thursday: none for Professor

extras may be added, TBD weekly

7:30–9:00 pm TA DJ, SMUDD 207

Friday: 12:00–3:00 pm

7:30–9:00 pm TA DJ, SMUDD 207

Match the sum formulas precisely.
Pattern find and check your guess too.
Happy short vacation!