

**Homework #17**Due **Wednesday, April 16th** in Gradescope by 11:59 pm ET

**Goal:** Exploring Estimating Values and Definite Integrals using the Alternating Series Estimation Theorem. Also some review of Interval and Radius of Convergence.

**FIRST:** Read through and understand the following Examples.

Ex: Use Series to Estimate  $\cos(1)$  with error less than  $\frac{1}{100}$ . Justify. Simplify.

First, recall  $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

Plugging in  $x = 1$ , we have

$$\cos(1) = \sum_{n=0}^{\infty} \frac{(-1)^n (1)^{2n}}{(2n)!} \stackrel{\text{Full Sum}}{=} 1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \dots$$

$$\stackrel{\text{Estimate}}{\approx} 1 - \frac{1}{2!} + \frac{1}{4!} = 1 - \frac{1}{2} + \frac{1}{24} = \frac{24}{24} - \frac{12}{24} + \frac{1}{24} = \boxed{\frac{13}{24}} \leftarrow \text{Estimate}$$

Using the Alternating Series Estimation Theorem (ASET), we can approximate the actual sum with only the first two terms as  $\frac{13}{24}$ , with error **At Most** the absolute value of the first neglected term,  $\frac{1}{6!} = \frac{1}{720} < \frac{1}{100}$  as desired.

Ex: Use Series to Estimate  $\int_0^1 x^3 \ln(1+x^3) dx$  with error less than  $\frac{1}{30}$ . Justify. Simplify.

$$\begin{aligned} \int_0^1 x^3 \ln(1+x^3) dx &= \int_0^1 x^3 \sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{n+1}}{n+1} dx = \int_0^1 x^3 \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+3}}{n+1} dx \\ &= \int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+6}}{n+1} dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+7}}{(n+1)(3n+7)} \Big|_0^1 = \frac{x^7}{1 \cdot 7} - \frac{x^{10}}{2 \cdot 10} + \frac{x^{13}}{3 \cdot 13} - \dots \Big|_0^1 \\ &= \frac{x^7}{7} - \frac{x^{10}}{20} + \frac{x^{13}}{39} - \dots \Big|_0^1 = \frac{1}{7} - \frac{1}{20} + \frac{1}{39} - \dots - (0 - 0 + 0 - \dots) \\ &\stackrel{\approx}{=} \frac{1}{7} - \frac{1}{20} = \frac{20}{140} - \frac{7}{140} = \boxed{\frac{13}{140}} \leftarrow \text{Estimate} \end{aligned}$$

Using the Alternating Series Estimation Theorem (ASET), we can approximate the actual sum with only the first two terms, and the error will be **At Most** the absolute value of the next (first neglected) term,  $\frac{1}{39} < \frac{1}{30}$  as desired.

**Continue on and Complete the next page of problems**

1. Use Series to Estimate  $\frac{1}{e}$  with error less than  $\frac{1}{20}$ . Justify.
2. Use Series to Estimate  $\frac{1}{e}$  with error less than  $\frac{1}{100}$ . Justify. (Can reuse work from 1)
3. Use Series to Estimate  $\frac{1}{e}$  with error less than  $\frac{1}{500}$ . Justify. (Can reuse work from 1)
4. Use Series to Estimate  $\sin(1)$  with error less than  $\frac{1}{1000}$ . Justify.
5. Use Series to Estimate  $e^{-\frac{1}{3}}$  with error less than  $\frac{1}{100}$ . Justify.
6. Use Series to Estimate  $\arctan\left(\frac{1}{2}\right)$  with error less than  $\frac{1}{100}$ . Justify.
7. Use Series to Estimate  $\int_0^1 x \ln(1+x^3) dx$  with error less than  $\frac{1}{20}$ . Justify.
8. Use Series to Estimate  $\int_0^1 x \sin(x^2) dx$  with error less than  $\frac{1}{1000}$ . Justify.

Review: Find the Interval and Radius of Convergence for each of the following.

9.  $\sum_{n=1}^{\infty} (n!)^2 (3x-7)^n$
10.  $\sum_{n=1}^{\infty} \frac{(-1)^n (5x-2)^n}{n^3 8^n}$
11.  $\sum_{n=1}^{\infty} \frac{(x-7)^n}{n! \sqrt{n}}$
12. New! Use Series to compute  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{1 + x - e^x}$ . Check answer with L'H Rule too.

# REGULAR OFFICE HOURS

**Monday: 12:00–3:00 pm**

6:00–7:30 pm TA Jude, SMUDD 207

**Tuesday: 1:00–4:00 pm**

6–7:30 pm TA Aaron, SMUDD 207

**Wednesday: 1:00–3:00 pm**

6–7:30 pm TA Jude, SMUDD 207

7:30–9:00 pm TA Aaron, SMUDD 207

**Thursday: none for Professor**

extras may be added, TBD weekly

7:30–9:00 pm TA DJ, SMUDD 207

**Friday: 12:00–3:00 pm**

7:30–9:00 pm TA DJ, SMUDD 207

Chase the fine details and make a full justification.

YES! Vacation coming!