

Homework #14Due **Wednesday, March 26th** in Gradescope by 11:59 pm ET

Goal: Exploring Convergence of Infinite Series. Focus on Absolute and Conditional Convergence...also using the Absolute Convergence Test. Finally... some review problems.

FIRST: Read through and understand the following Examples. Determine whether the Series is Absolutely Convergent, Conditionally Convergent, or Divergent.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 7}{n^7 + 2} \hookrightarrow \text{A.S. } \sum_{n=1}^{\infty} \frac{n^2 + 7}{n^7 + 2} \approx \sum_{n=1}^{\infty} \frac{n^2}{n^7} = \sum_{n=1}^{\infty} \frac{1}{n^5} \text{ Converges } p\text{-series } p = 5 > 1.$$

$$\text{Check: } \lim_{n \rightarrow \infty} \frac{\frac{n^2 + 7}{n^7 + 2}}{\frac{1}{n^5}} = \lim_{n \rightarrow \infty} \frac{n^7 + 7n^5}{n^7 + 2} \cdot \frac{\frac{1}{n^7}}{\frac{1}{n^7}} = \lim_{n \rightarrow \infty} \frac{1 + \frac{7}{n^2}}{1 + \frac{2}{n^7}} = 1 \text{ Finite/Non-zero}$$

The **Absolute Series** $\sum_{n=1}^{\infty} \frac{n^2 + 7}{n^7 + 2}$ also **Converges** by Limit Comparison Test (LCT).

Finally, the Original Series is Absolutely Convergent (A.C.) (by Definition).

$$\text{Ex: } \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{7n + 3} \hookrightarrow \text{A.S. } \sum_{n=1}^{\infty} \frac{1}{7n + 3} \approx \sum_{n=1}^{\infty} \frac{1}{n} \text{ Divergent Harmonic } p\text{-Series } p = 1$$

$$\text{Check: } \lim_{n \rightarrow \infty} \frac{\frac{1}{7n + 3}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{n}{7n + 3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{7 + \frac{3}{n}} = \frac{1}{7} \text{ Finite/Non-zero.}$$

Therefore, the **Absolute Series** also **Diverges** by Limit Comparison Test.

Now, we must examine the original alternating series with the Alternating Series Test.

• Isolate $b_n = \frac{1}{7n + 3} > 0$

• $\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{1}{7n + 3} \xrightarrow{\infty} 0$

• Terms Decreasing $b_{n+1} < b_n$ because $b_{n+1} = \frac{1}{7(n+1) + 3} = \frac{1}{7n + 10} < \frac{1}{7n + 3} = b_n$

Therefore, the **Original Series** **Converges** by the Alternating Series Test. Finally, we can conclude the Original Series is Conditionally Convergent (C.C.) (by Definition).

Now complete the following HW problems

Determine whether the given series is Absolutely Convergent, Conditionally Convergent or Divergent. Number 3 is Ratio Test, but use the AC and CC charts for 1, 2, 4, 5.

1. $\sum_{n=1}^{\infty} (-1)^n \frac{n^3 + 8}{n^8 + 3}$ 2. $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n + 1}$ 3. $\sum_{n=1}^{\infty} \frac{(-1)^n (\ln n) (2n)!}{n^n 2^{3n} (n!)}$

4. $\sum_{n=1}^{\infty} (-1)^n \frac{n + 1}{n^2}$ 5. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^3}{n^7 + 2}$

6. Write the statement of the Absolute Convergence Test.

7. Use the Absolute Convergent Test to show that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 + 1}$ Converges.

8. Use the Absolute Convergent Test to show that $\sum_{n=1}^{\infty} \frac{(-1)^n \sin^2 n}{n^8 + 2}$ Converges.

Review

9. Show that the Sequence $\left\{ \left(\frac{n}{n+1} \right)^n \right\}_{n=1}^{\infty}$ Converges to $\frac{1}{e}$.

10. Determine if the Series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^n$ Converges or Diverges.

11. Find the Sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n 5^{2n+1}}{2^{5n-1}}$.

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Jude, SMUDD 207

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Aaron, SMUDD 207

Wednesday: 1:00–3:00 pm

6–7:30 pm TA Jude, SMUDD 207

7:30–9:00 pm TA Aaron, SMUDD 207

Thursday: none for Professor

extras may be added, TBD weekly

7:30–9:00 pm TA DJ, SMUDD 207

Friday: 12:00–3:00 pm

7:30–9:00 pm TA DJ, SMUDD 207

This is the end of the material for the Exam 2. Material stops after
Section 11.6 Absolute Convergence Test and Ratio Test
and Absolute/Conditional Convergence.