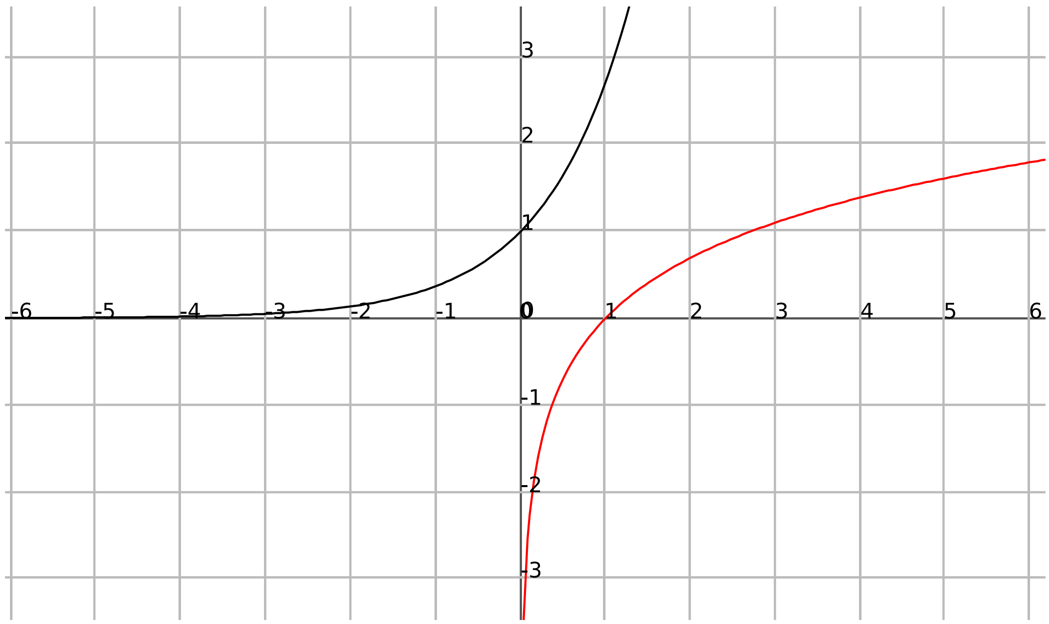
Overview of $e^x$ and $\ln x$ Functions

- **Graphs**




- **Function Properties**

**Natural Exponential Function** $y = e^x$

- Domain $= \mathbb{R}$
- Range $= (0, \infty)$
- $\lim_{x \to \infty} e^x = \infty$
- $\lim_{x \to -\infty} e^x = 0$
- $e$ chosen so that $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$
- $2 < e < 3$ and $e \approx 2.71828\ldots$
- $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$
- $e^0 = 1$ and $e^1 = e$

**Natural Logarithm Function** $y = \ln x$

- Domain $= (0, \infty)$
- Range $= \mathbb{R}$
- $\lim_{x \to \infty} \ln x = \infty$
- $\lim_{x \to 0^+} \ln x = -\infty$
- $y = \ln x$ is the inverse function for $y = e^x$
- $y = \ln x$ if and only if $e^y = x$
- $\begin{cases} \ln e^x = x & \text{for all } x \text{ in } \mathbb{R} \\ e^{\ln x} = x & \text{for all } x > 0 \end{cases}$
- $\ln 1 = 0$ and $\ln e = 1$
- $\ln 0$ is **UNDEFINED**

- **Algebraic Properties**

**Natural Exponential Function** $y = e^x$

- $e^x e^y = e^{x+y}$
- $e^{x-y} = \frac{e^x}{e^y}$
- $(e^x)^y = e^{xy}$
- $e^{(x^y)}$ cannot be simplified
- $\frac{1}{e^x} = e^{-x}$

**Natural Logarithm Function** $y = \ln x$

- $\ln(xy) = \ln x + \ln y$
- $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$
- $\ln(x^y) = y \ln x$
- $\ln(x+y)$ or $\frac{\ln x}{\ln y}$ cannot be simplified

- **Derivatives and Integrals**

**Natural Exponential Function** $y = e^x$

- $\frac{d}{dx} e^x = e^x$
- $\frac{d}{dx} e^{u(x)} = e^{u(x)} \frac{du}{dx} = e^{u(x)} u'(x)$
- $\int e^x \, dx = e^x + C$
- $\int e^{kx} \, dx = \frac{1}{k} e^{kx} + C \quad k \neq 0$ constant
  $k$-rule

**Natural Logarithm Function** $y = \ln x$

- $\frac{d}{dx} \ln x = \frac{1}{x}$
- $\frac{d}{dx} \ln u(x) = \frac{1}{u(x)} \frac{du}{dx} = \frac{u'(x)}{u(x)}$
- $\int \frac{1}{x} \, dx = \ln|x| + C$