Math 121 Midterm Exam #2 March 28, 2025

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\arctan(\sqrt{3})$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [27 Points] Compute the following **Improper** integrals. Simplify all answers. Justify.

(a) $\displaystyle\int_0^e x^4 \ln x \, dx$ (b) $\displaystyle\int_{-\infty}^1 \frac{1}{x^2 - 6x + 13} \, dx$ (c) $\displaystyle\int_{-7}^0 \frac{x + 15}{x^2 + 6x - 7} \, dx$

2. [10 Points] Use the Integral Test to determine if $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4}$ **Converges** or **Diverges**.

Note: You do **not** have to check the 3 pre-conditions for the Integral Test.

3. [25 Points] Determine whether each of the given series **Converges** or **Diverges**. Name any Convergence Test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{n^6}{\ln n}$ (b) $-4 - \frac{4}{2} - \frac{4}{3} - 1 - \frac{4}{5} - \frac{4}{6} - \dots$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n^6} + \frac{(-6)^n}{7^{2n}}$

4. [8 Points] Use the Absolute Convergence Test to show that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4+6}$ **Converges**.

5. [30 Points] Determine whether each of the given series is **Absolutely Convergent**, **Conditionally Convergent**, or **Divergent**. Name any Convergence Test(s) you use, and justify all of your work.

(a) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^4+6}{n^6+4}\right)$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n^n \cdot (2n)!}{6^n (n!)^3}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{6n+4}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 We have seen that the harmonic series is a **Divergent series whose terms do indeed approach zero** as $n \to \infty$. Show that the following series $\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right)$ is another series with this property.