1. Compute $\lim_{x\to 0} \dfrac{\cos(4x) - 1 - \arctan(4x) + 4x}{\ln(1-x) + \arcsin x}^{\left(\frac{0}{0}\right)} \overset{\text{L'H}}{=} \lim_{x\to 0} \dfrac{-4\sin(4x) - \dfrac{4}{1+16x^2} + 4}{-\dfrac{1}{1-x} + \dfrac{1}{\sqrt{1-x^2}}}^{\left(\frac{0}{0}\right)}$

$$\overset{\text{prep}}{=} \lim_{x\to 0} \frac{-4\sin(4x) - 4(1+16x^2)^{-1} + 4}{-(1-x)^{-1} + (1-x^2)^{-\frac{1}{2}}}^{\left(\frac{0}{0}\right)} \overset{\text{L'H}}{=} \lim_{x\to 0} \frac{-16\cos(4x) + 4(1+16x^2)^{-2}(32x)}{(1-x)^{-2}(-1) - \frac{1}{2}(1-x^2)^{-\frac{3}{2}}(-2x)}$$

$$\overset{\text{rewrite}}{=} \lim_{x\to 0} \frac{-16\cos(4x) + \dfrac{128\cancelto{0}{x}}{(1+16x^2)^2}}{-\dfrac{1}{(1-x)^2} + \dfrac{\cancelto{0}{x}}{(1-x)^{\frac{3}{2}}}} = \frac{-16+0}{-1+0} = \boxed{16}$$

2. $\lim_{x\to 0} \dfrac{1 - e^{-3x} - \arctan(3x)}{x^2}^{\left(\frac{0}{0}\right)} \overset{\text{L'H}}{=} \lim_{x\to 0} \dfrac{3e^{-3x} - \dfrac{3}{1+(3x)^2}}{2x}^{\left(\frac{0}{0}\right)}$

$$\overset{\text{prep}}{=} \lim_{x\to 0} \frac{3e^{-3x} - 3(1+9x^2)^{-1}}{2x}^{\left(\frac{0}{0}\right)} \overset{\text{L'H}}{=} \lim_{x\to 0} \frac{-9e^{-3x} + 3(1+9x^2)^{-2}(18x)}{2}$$

$$\overset{\text{rewrite}}{=} \lim_{x\to 0} \frac{-9e^{-3x} + \dfrac{54\cancelto{0}{x}}{(1+9x^2)^2}}{2} = \boxed{-\frac{9}{2}}$$

3. $\lim_{x\to\infty} \left(1 - \dfrac{2}{x^3}\right)^{7x^3} \overset{1^\infty}{=} e^{\lim_{x\to\infty} \ln\left(\left(1-\frac{2}{x^3}\right)^{7x^3}\right)} = e^{\lim_{x\to\infty} 7x^3 \ln\left(1-\frac{2}{x^3}\right)}$

$$\overset{\infty\cdot 0}{=} e^{\lim_{x\to\infty} \frac{7\ln\left(1-\frac{2}{x^3}\right)}{\frac{1}{x^3}}} \overset{\left(\frac{0}{0}\right)\text{L'H}}{=} e^{\lim_{x\to\infty} \frac{7\left(\frac{1}{1-\frac{2}{x^3}}\right)\left(\frac{6}{x^4}\right)}{-\frac{3}{x^4}}}$$

$$= e^{\lim_{x\to\infty} 7\left(\frac{1}{1-\frac{2}{x^3}}\right)\left(\frac{6}{\cancel{x^4}}\right)\cdot\left(-\frac{\cancel{x^4}}{3}\right)} = e^{\lim_{x\to\infty} 7\left(\frac{1}{1-\cancelto{0}{\frac{2}{x^3}}}\right)(-2)} = e^{7(1)(-2)} = \boxed{e^{-14}}$$

4. $$\lim_{x\to\infty}\left(\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}\right)^{x} \overset{1^\infty}{=} e^{\lim_{x\to\infty}\ln\left(\left(\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}\right)^{x}\right)}$$

$$= e^{\lim_{x\to\infty} x\ln\left(\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}\right)} \overset{\infty\cdot 0}{=} e^{\lim_{x\to\infty}\frac{\ln\left(\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}\right)^{\left(\frac{0}{0}\right)}}{\frac{1}{x}}}$$

$$\overset{\text{L'H}}{=} e^{\lim_{x\to\infty}\frac{\frac{1}{\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}}\left(\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\left(-\frac{1}{x^2}\right)+e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)\right)}{-\frac{1}{x^2}}}$$

$$= e^{\lim_{x\to\infty}\frac{\frac{1}{\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}}\left(\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}+e^{\frac{1}{x}}\right)}{1}} = e^{1(1+1)} = \boxed{e^2}$$

5. $$\lim_{x\to\infty}\left(1-\arctan\left(\frac{3}{x^4}\right)\right)^{x^4} \overset{1^\infty}{=} e^{\lim_{x\to\infty}\ln\left(\left(1-\arctan\left(\frac{3}{x^4}\right)\right)^{x^4}\right)}$$

$$= e^{\lim_{x\to\infty} x^4\ln\left(1-\arctan\left(\frac{3}{x^4}\right)\right)} \overset{\infty\cdot 0}{=} e^{\lim_{x\to\infty}\frac{\ln\left(1-\arctan\left(\frac{3}{x^4}\right)\right)^{\left(\frac{0}{0}\right)}}{\frac{1}{x^4}}}$$

$$\overset{\text{L'H}}{=} e^{\lim_{x\to\infty}\frac{\frac{1}{1-\arctan\left(\frac{3}{x^4}\right)}\left(-\frac{1}{1+\left(\frac{3}{x^4}\right)^2}\right)\left(-\frac{12}{x^5}\right)}{-\frac{4}{x^5}}} = e^{1(-1)(3)} = \boxed{e^{-3}}$$

6. $\lim_{x\to0^+} x^3\ln x \overset{0\cdot(-\infty)}{=} \lim_{x\to0^+} \dfrac{\ln x}{\frac{1}{x^3}}^{\left(\frac{-\infty}{\infty}\right)} \overset{\text{L'H}}{=} \lim_{x\to0^+} \dfrac{\frac{1}{x}}{-\frac{3}{x^4}} = \lim_{x\to0^+} \dfrac{1}{x}\left(-\dfrac{x^4}{3}\right) = \lim_{x\to0^+} -\dfrac{x^3}{3} = \boxed{0}$

7. $\int \ln x\ dx \int \ln x \cdot 1\ dx = x\ln x - \int \cancel{x}\left(\dfrac{1}{\cancel{x}}\right)\ dx = x\ln x - \int 1\ dx = \boxed{x\ln x - x + C}$

I.B.P. $\boxed{\begin{array}{ll} u=\ln x & dv = 1dx \\ du = \frac{1}{x}dx & v = x \end{array}}$

8. $\int \arctan x\ dx = \int \arctan x \cdot 1\ dx = x\arctan x - \int \dfrac{x}{1+x^2}\ dx$

$= x\arctan x - \dfrac{1}{2}\int \dfrac{1}{w}\ dw = x\arctan x - \dfrac{1}{2}\ln|w| + C = \boxed{x\arctan x - \dfrac{1}{2}\ln|1+x^2| + C}$

I.B.P. $\boxed{\begin{array}{ll} u=\arctan x & dv = 1dx \\ du = \frac{1}{1+x^2}dx & v = x \end{array}}$ w-sub $\boxed{\begin{array}{l} w = 1+x^2 \\ dw = 2xdx \\ \frac{1}{2}dw = xdx \end{array}}$

9. $\int \arcsin x\ dx = \int \arcsin x \cdot 1\ dx = x\arcsin x - \int \dfrac{x}{\sqrt{1-x^2}}\ dx$

$= x\arcsin x + \dfrac{1}{2}\int \dfrac{1}{\sqrt{w}}\ dw = x\arcsin x + \dfrac{1}{2}\int w^{-\frac{1}{2}}\ dw = x\arcsin x + \dfrac{\cancel{1}}{\cancel{2}}\left(\dfrac{w^{\frac{1}{2}}}{\frac{\cancel{1}}{\cancel{2}}}\right) + C$

$= \boxed{x\arcsin x + \sqrt{1-x^2} + C}$

I.B.P. $\boxed{\begin{array}{ll} u=\arcsin x & dv = 1dx \\ du = \frac{1}{\sqrt{1-x^2}}dx & v = x \end{array}}$ w-sub $\boxed{\begin{array}{l} w = 1-x^2 \\ dw = -2xdx \\ -\frac{1}{2}dw = xdx \end{array}}$

10. $\int \ln\left(x^2+9\right)\ dx = \int \ln\left(x^2+9\right)\cdot 1\ dx = x\ln(x^2+9) - 2\int \dfrac{x^2}{x^2+9}\ dx$

$= x\ln(x^2+9) - 2\int \dfrac{x^2+9-9}{x^2+9}\ dx = x\ln(x^2+9) - 2\int \dfrac{\cancel{x^2+9}}{\cancel{x^2+9}} - \dfrac{9}{x^2+9}\ dx$

$= x\ln(x^2+9) - 2\int 1 - \dfrac{9}{x^2+9}\ dx = x\ln(x^2+9) - 2\left(x - 9\int \dfrac{1}{x^2+9}\ dx\right)$

$$= x\ln(x^2+9) - 2\left(x - 9\left(\frac{1}{3}\right)\arctan\left(\frac{x}{3}\right)\right) + C = \boxed{x\ln(x^2+9) - 2x + 6\arctan\left(\frac{x}{3}\right) + C}$$

I.B.P. $\boxed{\begin{array}{ll} u = \ln(x^2+9) & dv = 1dx \\ & \\ du = \dfrac{2x}{x^2+9}dx & v = x \end{array}}$

11. Show that $\displaystyle\int_0^{\sqrt{3}} x\arctan x\ dx = \frac{x^2}{2}\cdot\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} \frac{x^2}{1+x^2}\ dx$

$$= \frac{x^2}{2}\cdot\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} \frac{x^2+1-1}{1+x^2}\ dx = \frac{x^2}{2}\cdot\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} \cancelto{1}{\frac{x^2+1}{1+x^2}} - \frac{1}{1+x^2}\ dx$$

$$= \frac{x^2}{2}\cdot\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}\left(x - \arctan x\right)\Big|_0^{\sqrt{3}} = \frac{x^2}{2}\cdot\arctan x - \frac{x}{2} + \frac{1}{2}\arctan x\Big|_0^{\sqrt{3}}$$

$$= \frac{3}{2}\cancelto{\frac{\pi}{3}}{\arctan\sqrt{3}} - \frac{\sqrt{3}}{2} + \frac{1}{2}\cancelto{\frac{\pi}{3}}{\arctan\sqrt{3}} - \left(\cancelto{0}{0\arctan 0} - 0 + \frac{1}{2}\cancelto{0}{\arctan 0}\right)$$

$$= \frac{4}{2}\left(\frac{\pi}{3}\right) - \frac{\sqrt{3}}{2} = \boxed{\frac{2\pi}{3} - \frac{\sqrt{3}}{2}}$$

I.B.P. $\boxed{\begin{array}{ll} u = \arctan x & dv = xdx \\ & \\ du = \dfrac{1}{1+x^2}dx & v = \dfrac{x^2}{2} \end{array}}$ w-sub $\boxed{\begin{array}{l} w = 1+x^2 \\ \\ dw = 2xdx \\ \\ \frac{1}{2}dw = xdx \end{array}}$

12. Show that $\displaystyle\int_1^{e^4} \frac{\ln x}{\sqrt{x}}\ dx = \int_1^{e^4} \ln x\cdot x^{-\frac{1}{2}}\ dx = 2\sqrt{x}\ln x\Big|_1^{e^4} - 2\int_1^{e^4} \frac{\sqrt{x}}{x}\ dx$

$$= 2\sqrt{x}\ln x\Big|_1^{e^4} - 2\int_1^{e^4} x^{-\frac{1}{2}}\ dx = 2\sqrt{x}\ln x\Big|_1^{e^4} - 2\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right)\Bigg|_1^{e^4}$$

$$= 2\sqrt{x}\ln x - 4\sqrt{x}\Big|_1^{e^4} = 2\cancelto{e^2}{\sqrt{e^4}}\cancelto{4}{\ln(e^4)} - 4\cancelto{e^2}{\sqrt{e^4}} - \left(2\cancelto{0}{\ln 1} - 4\sqrt{1}\right) = 8e^2 - 4e^2 + 4 = \boxed{4e^2 + 4}$$

I.B.P. $\boxed{\begin{array}{ll} u = \ln x & dv = x^{-\frac{1}{2}}dx \\ & \\ du = \dfrac{1}{x}dx & v = 2\sqrt{x} \end{array}}$