Math 121, Section(s) 01, Spring 2024

Homework #4

Due **Friday, February 9th** in Gradescope by 11:59 pm ET

Goal: Reviewing Inverse Trigonometric Functions and Limits (no L'Hopital's Rule yet)

FIRST: Read through and understand the following Examples.

Ex:

$$\int_e^{e^3} \frac{1}{x\left(3+(\ln x)^2\right)}\,dx = \int_1^3 \frac{1}{3+u^2}\,du \overset{\text{a-rule}}{=} \frac{1}{\sqrt{3}}\arctan\left(\frac{u}{\sqrt{3}}\right)\Bigg|_1^3$$

$$= \frac{1}{\sqrt{3}}\left(\arctan\left(\frac{3}{\sqrt{3}}\right) - \arctan\left(\frac{1}{\sqrt{3}}\right)\right) \quad \left(\frac{3}{\sqrt{3}} \to \sqrt{3}\right)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}\left(\frac{2\pi}{6}-\frac{\pi}{6}\right) = \boxed{\frac{\pi}{6\sqrt{3}}}$$

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}} \qquad \boxed{\begin{aligned} x = e &\Rightarrow u = \ln e = 1 \\ x = e^3 &\Rightarrow u = \ln e^3 = 3 \end{aligned}}$$

Ex:

$$\int \frac{e^{3x}}{4+e^{3x}}\,dx = \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C = \boxed{\frac{1}{3}\ln\left|4+e^{3x}\right| + C}$$

$$\boxed{\begin{aligned} u &= 4+e^{3x} \\ du &= 3e^{3x}\,dx \\ \frac{1}{3}du &= e^{3x}\,dx \end{aligned}}$$

Ex:

$$\int \frac{e^{3x}}{4+e^{6x}}\,dx = \int \frac{e^{3x}}{4+\left(e^{3x}\right)^2}\,dx \quad \text{look for the perfect square to ready for arctan rule}$$

$$\frac{1}{3}\int \frac{1}{4+u^2}\,du \overset{\text{a-rule}}{=} \frac{1}{3}\left(\frac{1}{2}\arctan\left(\frac{u}{2}\right)\right) + C = \boxed{\frac{1}{6}\arctan\left(\frac{e^{3x}}{2}\right) + C}$$

$$\boxed{\begin{aligned} u &= e^{3x} \\ du &= 3e^{3x}\,dx \\ \frac{1}{3}du &= e^{3x}\,dx \end{aligned}}$$

Next, Compute each of the following Integrals. Simplify.

1. $\displaystyle\int_2^{2\sqrt{3}} \frac{1}{\sqrt{16-x^2}}\,dx$

2. $\displaystyle\int_0^{\ln 3} \frac{e^x}{3+e^{2x}}\,dx$

3. $\displaystyle\int_0^{\ln\sqrt{3}} \frac{e^x}{\sqrt{4-e^{2x}}}\,dx$

4. $\displaystyle\int_4^{4\sqrt{3}} \frac{1}{16+x^2}\,dx$

5. $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$

6. $\displaystyle\int \frac{x^2}{x^2+4}\,dx$

7. $\displaystyle\int \frac{2x^2+5}{x^2+1}\,dx$

8. $\displaystyle\int \frac{1}{(1+x^2)\,(5+(\arctan x)^2)}\,dx$

9. $\displaystyle\int_3^{9} \frac{1}{\sqrt{x}\,(x+9)}\,dx$

10. $\displaystyle\int \frac{x^2+x+1}{x^2+4}\,dx$

Compute each of the following Limits. Simplify. Use arrows to justify the size arguments.

11. $\displaystyle\lim_{x\to 5^+} \frac{1}{x-5}$

12. $\displaystyle\lim_{x\to 5^-} \frac{1}{x-5}$

13. $\displaystyle\lim_{x\to 8^+} \ln|x-8|$

14. $\displaystyle\lim_{x\to 8^-} \ln|x-8|$

15. $\displaystyle\lim_{x\to 3^+} e^{\frac{2}{x-3}}$

16. $\displaystyle\lim_{x\to 3^-} e^{\frac{2}{x-3}}$

17. $\displaystyle\lim_{x\to\infty} \ln\left(1-\arctan\left(\frac{5}{x^4}\right)\right)$

18. $\displaystyle\lim_{x\to\infty} \ln\left(\frac{\pi}{2}-\arctan x\right)$

19. $\displaystyle\lim_{x\to 4^-} \ln|\ln|x-4||$

20. $\displaystyle\lim_{x\to 0^+} \arctan\left(\frac{\ln x}{5}\right)$

21. Present two different methods to Prove that $\displaystyle\int \frac{1}{4+x^2}\,dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

<u>REGULAR OFFICE HOURS</u>

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Gretta, SMUDD 208

Tuesday: 1:00–4:00 pm

7:30–9:00 pm TA Aidee, SMUDD 208

9–10:30 pm TA Natalie, SMUDD 208

Wednesday: 1:00-3:00 pm

7:30–9:00 pm TA Gretta, SMUDD 208

Thursday: none for Professor

7:30–9:00 pm TA Aidee, SMUDD 208

9:00–10:30 pm TA Natalie, SMUDD 208

Friday: 12:00–2:00 pm

• Please stop by for help! Try to attend at least one office hour for me and at least one for the Math Fellows each week.

• You can also find help at the Math Fellow (Aidee, James, Natalie, Gretta, or Admire) sessions.