Math 121, Section(s) 01, Spring 2024

Homework #3

Due **Friday, February 7th** in Gradescope by 11:59 pm ET

Goal: Solidying Calculus for Inverse Sine and Inverse Tangent.

FIRST: Read through and understand the following two Derivative proofs.

Ex: **PROVE** that $\dfrac{d}{dx}\arctan x = \dfrac{1}{1+x^2}$

Proof: Let $y = \arctan x$ Looking to solve for $\dfrac{dy}{dx}$

Invert $\tan y = x$

Differentiate $\dfrac{d}{dx}(\tan y) = \dfrac{d}{dx}(x)$

$$\sec^2 y \cdot \frac{dy}{dx} = 1$$

Solve $\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y} = \dfrac{1}{1+\tan^2 y} = \dfrac{1}{1+(\tan y)^2} = \dfrac{1}{1+x^2}$

Ex: **PROVE** that $\dfrac{d}{dx}\arcsin x = \dfrac{1}{\sqrt{1-x^2}}$

Proof: Let $y = \arcsin x$ Looking to solve for $\dfrac{dy}{dx}$

Invert $\sin y = x$

Differentiate $\dfrac{d}{dx}(\sin y) = \dfrac{d}{dx}(x)$

$$\cos y \cdot \frac{dy}{dx} = 1$$

Solve $\dfrac{dy}{dx} = \dfrac{1}{\cos y} = \dfrac{1}{\sqrt{1-\sin^2 y}} = \dfrac{1}{\sqrt{1-(\sin y)^2}} = \dfrac{1}{\sqrt{1-x^2}}$

Continue to NEXT Page for HW problems.

Differentiate the following functions for 1-4. Simplify.

1. $f(x) = \tan^{-1}(x^2)$
2. $f(x) = (\tan^{-1}(x))^2$
3. $y = x\sin^{-1}x + \sqrt{1-x^2}$
4. $f(x) = \ln\left(1 - \arcsin\left(\frac{2}{x^4}\right)\right)$

5. Find the value of the expression $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$

6. Simplify the expression $\sin(\tan^{-1}x)$

7. Compute the Second Derivative for $f(x) = \arctan(2x)$

8. Compute the Second Derivative for $f(x) = \arcsin(6x)$

9. **Prove** that $\frac{d}{dx}\sin^{-1}(3x) = \frac{3}{\sqrt{1-9x^2}}$

10. **Prove** that $\frac{d}{dx}\tan^{-1}(5x) = \frac{5}{1+25x^2}$

11. Use Integration to **Justify** that $\int \frac{1}{3+x^2}\,dx = \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C$

Compute each of the following Integrals. Simplify.

12. $\int \frac{x^2}{x^2+1}\,dx$
13. $\int \frac{x+1}{x^2+1}\,dx$
14. $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{8}{1+x^2}\,dx$
15. $\int_0^{\frac{1}{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$
16. $\int \frac{1}{\sqrt{1-x^2}\cdot\sin^{-1}x}\,dx$
17. $\int_1^3 \frac{1}{\sqrt{x}\,(1+x)}\,dx$
18. $\int_0^{\ln 3} \frac{e^x}{1+e^x}\,dx$
19. $\int_0^{\frac{1}{2}\ln 3} \frac{e^x}{1+e^{2x}}\,dx$
20. $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}\,dx$
21. $\int_3^{3\sqrt{3}} \frac{1}{\sqrt{36-x^2}} + \frac{1}{9+x^2}\,dx$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Gretta, SMUDD 208

Tuesday: 1:00–4:00 pm

7:30–9:00 pm TA Aidee, SMUDD 208

9–10:30 pm TA Natalie, SMUDD 208

Wednesday: 1:00-3:00 pm

7:30–9:00 pm TA Gretta, SMUDD 208

Thursday: none for Professor

7:30–9:00 pm TA Aidee, SMUDD 208

9:00–10:30 pm TA Natalie, SMUDD 208

Friday: 12:00–2:00 pm

• Please do not wait until the last night to start.

• Please stop by for help! Please try the homework before you come by though. Final Answer keys are posted on the webpage. Please do **not** look at them unless you have completed the problems. **They are not a replacement for my help or your understanding.**

• You can also find help at the Math Fellow (Aidee, Natalie, or Gretta) sessions.