Math 121, Section(s) 01, Spring 2024

Homework #2

Due **Friday, February 2nd** in Gradescope by 11:59 pm ET

Goal: Review of Limits, Derivatives and Integrals for Exponentials and Logarithms. Plenty of help in Office Hours!

FIRST: Read through and understand the following Examples.

Think about the graph of $y = \ln x$. We know that $\boxed{\lim_{x\to 0^+} \ln x = \lim_{x\to 0^+} \ln x \overset{0^+}{\to} = -\infty}$. Learn this!

Warning: Do **not** write $\ln 0$; it is undefined.

Ex: $\lim_{x\to 5^+} \ln(x-5) = \lim_{x\to 5^+} \ln(x-5 \overset{5^+}{\to} \overset{0^+}{\to}) = -\infty$ The arrows help justify the size argument(s).

Ex: $\lim_{x\to 8^-} \ln|x-8| = \lim_{x\to 8^-} \ln|x-8 \overset{8^-}{\to} \overset{0^+}{\to} \overset{0^-}{\to}| = -\infty$

Ex: $\displaystyle\int \frac{(3-\sqrt{x})(1+2\sqrt{x})}{x^2}\,dx = \int \frac{3+6\sqrt{x}-\sqrt{x}-2x}{x^2}\,dx = \int \frac{3+5\sqrt{x}-2x}{x^2}\,dx$

$\displaystyle= \int \frac{3}{x^2}+\frac{5\sqrt{x}}{x^2}-\frac{2x}{x^2}\,dx = \int \frac{3}{x^2}+\frac{5}{x^{\frac{3}{2}}}-\frac{2}{x}\,dx = \int \frac{3}{x^2}+\frac{5}{x^{\frac{3}{2}}}-\frac{2}{x}\,dx$

$\overset{\text{prep}}{=} \displaystyle\int 3x^{-2}+5x^{-\frac{3}{2}}-\frac{2}{x}\,dx = -3x^{-1}+5(-2)x^{-\frac{1}{2}}-2\ln|x|+C = \boxed{-\frac{3}{x}-\frac{10}{\sqrt{x}}-2\ln|x|+C}$

Ex: $\displaystyle\int_{\ln 3}^{\ln 8} \frac{e^x}{\sqrt{1+e^x}}\,dx = \int_4^9 \frac{1}{\sqrt{u}}\,du = \int_4^9 u^{-\frac{1}{2}}\,du = 2\sqrt{u}\Big|_4^9 = 2\sqrt{9}^{3} - 2\sqrt{4}^{2} = 6-4 = \boxed{2}$

Here $\boxed{\begin{aligned} u &= 1+e^x \\ du &= e^x\,dx \end{aligned}}$ and $\boxed{\begin{aligned} x=\ln 3 &\implies u = 1+e^{\ln 3} = 1+3 = 4 \\ x=\ln 8 &\implies u = 1+e^{\ln 8} = 1+8 = 9 \end{aligned}}$

Ex: $\displaystyle\int_1^2 \frac{1}{3-5x}\,dx = -\frac{1}{5}\int_{-2}^{-7} \frac{1}{u}\,du = -\frac{1}{5}\ln|u|\Big|_{-2}^{-7} = -\frac{1}{5}\left(\ln|-7|-\ln|-2|\right) = \boxed{-\frac{1}{5}\ln\left(\frac{7}{2}\right)}$

Here $\boxed{\begin{aligned} u &= 3-5x \\ du &= -5\,dx \\ -\tfrac{1}{5}du &= dx \end{aligned}}$ and $\boxed{\begin{aligned} x=1 &\implies u = 3-5 = -2 \\ x=2 &\implies u = 3-10 = -7 \end{aligned}}$

Next, Complete the following Homework problems.

Differentiate the following functions. Simplify.

1. $f(x) = e^5$
2. $f(x) = e^x + x^e$
3. $y = \dfrac{1 - e^{2x}}{1 + e^{2x}}$
4. $f(x) = e^{\sin(2x)} + \sin\left(e^{2x}\right)$
5. $y = e^{\sqrt{x}}$
6. $y = x^2 e^{-\frac{1}{x}}$
7. $y = \ln\left(1 + e^{3x}\right)$
8. $f(x) = \ln\left(\dfrac{1}{x}\right) + \dfrac{1}{\ln x}$

9. Express the quantity as a single logarithm. Simplify.

$$\frac{1}{3}\ln[(x+2)^3] + \frac{1}{2}[\ln x - \ln[(x^2+3x+2)^2]]$$

Solve each of the following equations for x:

10. $e^{7-4x} = 6$

11. $\ln(3x - 10) = 2$

Evaluate each of the following Limits. Justify the size argument(s) using arrows.

12. $\lim\limits_{x \to 2^-} \ln|x - 2|$

13. $\lim\limits_{x \to 3^+} \ln(x^2 - 9)$

Evaluate each of the following Integrals. Simplify. Justify.

14. $\displaystyle\int e^x + x^e \, dx$
15. $\displaystyle\int_0^{\ln 4} \frac{1}{e^{2x}} \, dx$
16. $\displaystyle\int \frac{(1 + e^x)^2}{e^x} \, dx$
17. $\displaystyle\int \left(e^x + e^{-x}\right)^2 \, dx$
18. $\displaystyle\int \frac{e^x}{1 + e^x} \, dx$
19. $\displaystyle\int_2^3 \frac{1}{5 - 4x} \, dx$
20. $\displaystyle\int_e^{e^3} \frac{4}{x(\ln x)^2} \, dx$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

Tuesday: 1:00–4:00 pm

Wednesday: 1:00-3:00 pm

Friday: 12:00–2:00 pm

Math Fellow evening TA Help Hours TBD soon

• Office Hours are open to everyone. Please feel welcome whether you have lots of questions or just one question. Just stop by. :-) Working on your calculus assignment can be fun! You are encouraged to make fully engaged visits to office hours **each** week. I hope that you come hang out at many help sessions.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.