Math 121, Section(s) 01, Spring 2024 Homework #16 Due Friday, April 12th in Gradescope by 11:59 pm ET

Goal: Exploring more of the Relationship between Power Series and functions, including Differentiation and Integration of Power Series. Also *substitution* into a known MacLaurin Series. Also SUMS which are not Geometric.

Find the Series Representation for the following functions using *substitution* and determine the Radius of Convergence R. Simplify.

1.
$$\frac{1}{1+x^2}$$
 2. $\frac{x^2}{x^4+16}$ 3. $x^3 \cos(x^2)$ 4. $5x^2 \sin(5x)$

5.
$$\frac{d}{dx} (x^3 \arctan(7x))$$
 6. $\int x^3 \arctan(7x) dx$ 7. $\frac{d}{dx} x^2 \ln(1+6x)$ 8. $\int x^4 e^{-x^3} dx$

9. Find the Series Representation for $f(x) = \frac{1}{(1+x)^2}$

Hint:
$$\frac{1}{(1+x)^2} = \frac{d}{dx} \left(-\frac{1}{1+x} \right)^{PS?} = \dots$$

10. Prove the Power Series Representation formula for $\arctan x$, as shown in class. Yes, show that C = 0.

11. Find Series Representation for $\ln(5-x)$. Solve for C and the Radius R.

Hint:
$$\ln(5-x) = \int \frac{-1}{5-x} dx = \int \frac{-1}{5\left(1-\frac{x}{5}\right)} dx = -\frac{1}{5}\int \frac{1}{1-\frac{x}{5}} dx = \dots$$

12. Find the MacLaurin Series for $f(x) = e^{-2x}$ using two different methods. **First**, using the *Definition* of the MacLaurin Series ("Chart Method"). **Second**, use Substitution into a known series. Your answers should be in Sigma notation.

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13. You do **not** need to state the Radius. Answers should be in Sigma notation $\sum_{n=0}^{\infty}$ here.

You may use the fact that $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ without extra justification.

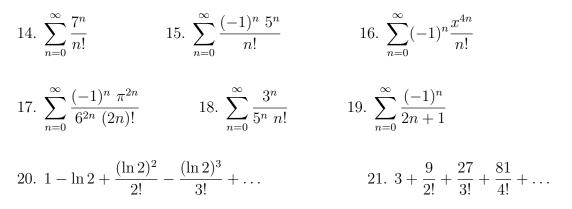
(a) Use the Definition ("Chart Method") to compute the MacLaurin Series for $F(x) = \cos x$.

(b) Use Differentiation to compute the Series for $F(x) = \cos x$.

(c) Use Integration to compute the Series for $F(x) = \cos x$.

Hints: yes, you should solve for +C. yes, C should equal 1. Show why C = 1.

Find the Sum of each of the following Series, which do converge.



REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Gretta, SMUDD 208

Tuesday: 1:00–4:00 pm

7:30–9:00 pm TA Aidee, SMUDD 208

9–10:30 pm TA Natalie, SMUDD 208

Wednesday: 1:00-3:00 pm

7:30–9:00 pm TA Gretta, SMUDD 208 Thursday: none for Professor 7:30–9:00 pm TA Aidee, SMUDD 208 9:00–10:30 pm TA Natalie, SMUDD 208 Friday: 12:00–2:00 pm

> Pay careful attention to details here. Manipulating power series requires a balance of memory and technical skill.