Math 121, Section(s) 01, Spring 2024

Homework #14

Due **Friday, March 27th** in Gradescope by 11:59 pm ET

Goal: Exploring Convergence of Infinite Series. Focus on Absolute and Conditional Convergence...also using the Absolute Convergence Test. Finally... some review problems.

FIRST: Read through and understand the following Examples. Determine whether the Series is Absolutely Convergent, Conditionally Convergent, or Divergent.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2+7}{n^7+2} \hookrightarrow \text{A.S.} \sum_{n=1}^{\infty} \frac{n^2+7}{n^7+2} \approx \sum_{n=1}^{\infty} \frac{n^2}{n^7} = \sum_{n=1}^{\infty} \frac{1}{n^5} \text{ Converges } p\text{-series } p = 5 > 1.$$

Check: $\displaystyle\lim_{n\to\infty} \frac{\frac{n^2+7}{n^7+2}}{\frac{1}{n^5}} = \lim_{n\to\infty} \frac{n^7+7n^5}{n^7+2} \cdot \frac{\frac{1}{n^7}}{\frac{1}{n^7}} = \lim_{n\to\infty} \frac{1+\frac{7}{n^2}}{1+\frac{2}{n^7}} = 1$ Finite/Non-zero

The **Absolute Series** $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+7}{n^7+2}$ also **Converges** by Limit Comparison Test (LCT).

Finally, the Original Series is Absolutely Convergent (A.C.) (by Definition).

Ex: $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{7n+3} \hookrightarrow \text{A.S.} \sum_{n=1}^{\infty} \frac{1}{7n+3} \approx \sum_{n=1}^{\infty} \frac{1}{n}$ Divergent Harmonic p-Series $p = 1$

Check: $\displaystyle\lim_{n\to\infty} \frac{\frac{1}{7n+3}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{7n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{7+\frac{3}{n}} = \frac{1}{7}$ Finite/Non-zero.

Therefore, the **Absolute Series** also **Diverges** by Limit Comparison Test.

Now, we must examine the original alternating series with the Alternating Series Test.

- Isolate $b_n = \dfrac{1}{7n+3} > 0$
- $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{7n+3} = \frac{1}{\infty} = 0$
- Terms Decreasing $\dfrac{1}{b_{n+1}} < \dfrac{1}{b_n}$ because $b_{n+1} = \dfrac{1}{7(n+1)+3} = \dfrac{1}{7n+10} < \dfrac{1}{7n+3} = b_n$

Therefore, the **Original Series Converges** by the Alternating Series Test. Finally, we can conclude the Original Series is Conditionally Convergent (C.C.) (by Definition).

Now complete the following HW problems

Determine whether the given series is Absolutely Convergent, Conditionally Convergent or Divergent. Number 3 is Ratio Test, but use the AC and CC charts for 1, 2, 4, 5.

1. $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n^3+8}{n^8+3}$ 2. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{5n+1}$ 3. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\ (\ln n)\ (2n)!}{n^n\ 2^{3n}\ (n!)}$

4. $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n+1}{n^2}$ 5. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^3}{n^7+2}$

6. Write the statement of the Absolute Convergence Test.

7. Use the Absolute Convergent Test to show that $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3+1}$ Converges.

8. Use the Absolute Convergent Test to show that $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\sin^2 n}{n^8+2}$ Converges.

Review

9. Show that the Sequence $\displaystyle\left\{\left(\frac{n}{n+1}\right)^n\right\}_{n=1}^{\infty}$ Converges to $\frac{1}{e}$.

10. Determine if the Series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^n$ Converges or Diverges.

11. Find the Sum of the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\ 5^{2n+1}}{2^{5n-1}}$.

<u>REGULAR OFFICE HOURS</u>

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Gretta, SMUDD 208

Tuesday: 1:00–4:00 pm

7:30–9:00 pm TA Aidee, SMUDD 208

9–10:30 pm TA Natalie, SMUDD 208

Wednesday: 1:00-3:00 pm

7:30–9:00 pm TA Gretta, SMUDD 208

Thursday: none for Professor

7:30–9:00 pm TA Aidee, SMUDD 208

9:00–10:30 pm TA Natalie, SMUDD 208

Friday: 12:00–2:00 pm

This is the end of the material for the Exam 2. Material stops after Section 11.6 Absolute Convergence Test and Ratio Test and Absolute/Conditional Convergence.