

Homework #13

Due **Friday, March 15th** in Gradescope by 11:59 pm ET

Goal: Exploring Convergence of Infinite Series. Focus on Alternating Series Test, and Ratio Test. We will also focus on fluency of training, using multiple tests.

FIRST: Read through and understand the following Examples. Determine whether the given Series Converges Absolutely or Diverges. Justify.

Ex: $\sum_{n=1}^{\infty} \frac{n^n}{n! \cdot 2^n}$ Try Ratio Test:

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{(n+1)^{n+1}}{(n+1)! 2^{n+1}}}{\frac{n^n}{n! \cdot 2^n}} \right| = \lim_{n \rightarrow \infty} \frac{(n+1)^{n+1}}{n^n} \cdot \frac{n!}{(n+1)!} \cdot \frac{2^n}{2^{n+1}}$$

$$= \lim_{n \rightarrow \infty} \frac{(n+1)^n \cancel{(n+1)}}{n^n} \cdot \frac{\cancel{n!}}{(n+1)\cancel{n!}} \cdot \frac{2^n}{2^n \cdot 2} = \lim_{n \rightarrow \infty} \left(\frac{n+1}{n} \right)^n \cdot \left(\frac{1}{2} \right) = \frac{e}{2} > 1$$

The Original Series Diverges by the Ratio Test.

Ex: $\sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{e^{2n} \cdot n! \cdot n^n}$ Try Ratio Test:

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{(-1)^{n+1} (2(n+1))!}{e^{2(n+1)} (n+1)! (n+1)^{n+1}}}{\frac{(-1)^n (2n)!}{e^{2n} n! n^n}} \right|$$

$$= \lim_{n \rightarrow \infty} \frac{(2n+2)!}{(2n)!} \cdot \frac{n^n}{(n+1)^{n+1}} \cdot \frac{e^{2n}}{e^{2n+2}} \cdot \frac{n!}{(n+1)!}$$

$$= \lim_{n \rightarrow \infty} \frac{(2n+2)(2n+1)(2n)!}{(2n)!} \cdot \frac{n^n}{(n+1)^n (n+1)} \cdot \frac{e^{2n}}{e^{2n} e^2} \cdot \frac{\cancel{n!}}{(n+1)\cancel{n!}}$$

$$= \lim_{n \rightarrow \infty} \frac{(2\cancel{(n+1)})(2n+1)}{(\cancel{n+1})(n+1)} \cdot \left(\frac{n}{n+1} \right)^n \cdot \frac{1}{e^2}$$

$$= \lim_{n \rightarrow \infty} \frac{2n+1}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \cdot \left(\frac{2}{e^3} \right) = \lim_{n \rightarrow \infty} \frac{2 + \frac{1}{n}}{1 + \frac{1}{n}} \cdot \left(\frac{2}{e^3} \right) = 2 \left(\frac{2}{e^3} \right) = \frac{4}{e^3} < 1$$

The original series is Absolutely Convergent (A.C.) by the Ratio Test.

Now complete the following HW problems

1. Consider $\sum_{n=1}^{\infty} \frac{n+1}{n^2+4n+7}$. Use **two** Different methods, namely the Integral Test and the Limit Comparison Test, to prove that this series Diverges. *You can skip checking the Integral Test preconditions here this time. yay!*

2. Determine if the given Alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n+1}$ Converges or Diverges.

Determine if the given series is Absolutely Convergent or Divergent.

3. $\sum_{n=1}^{\infty} \frac{n}{5^n}$ 4. $\sum_{n=1}^{\infty} \frac{(-3)^n}{(2n+1)!}$ 5. $\sum_{n=1}^{\infty} \frac{n!}{100^n}$

6. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ 7. $\sum_{n=1}^{\infty} \frac{n^{100} 100^n}{n!}$ 8. $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

9. Consider the series $\sum_{n=1}^{\infty} \frac{\ln n}{n}$.

(a) Show that n^{th} Term Divergence Test is **Inconclusive**.

(b) Show that the Ratio Test is **Inconclusive**.

(c) Show that the series Diverges using the Integral Test. Skip checking the 3 preconditions here. **Note:** This is an example where the terms approach 0 but the series Diverges.

10. Prove that $\sum_{n=1}^{\infty} \frac{6}{n^6}$ is Convergent by using the Limit Comparison Test.

Note that this work will be a sample proof of the fact that *Constant multiple of a Convergent series is Convergent*.

11. Show that $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ Diverges using **two** Different methods.

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Gretta, SMUDD 208

Tuesday: 1:00–4:00 pm

7:30–9:00 pm TA Aidee, SMUDD 208

9–10:30 pm TA Natalie, SMUDD 208

Wednesday: 1:00–3:00 pm

7:30–9:00 pm TA Gretta, SMUDD 208

Thursday: none for Professor

7:30–9:00 pm TA Aidee, SMUDD 208

9:00–10:30 pm TA Natalie, SMUDD 208

Friday: 12:00–2:00 pm

Train your Convergence Tests Daily Happy Spring Break!