Math 121, Section(s) 01, Spring 2024

Homework #12

Due **Wednesday, March 13th** in Gradescope by 11:59 pm ET

Goal: Exploring Convergence of Infinite Series. Focus on Integral Test, p-series, Comparison and Limit Comparison Test. We will also focus on fluency of training, using multiple tests.

FIRST: Read through and understand the following Examples. Determine whether the given Series Converges or Diverges. Justify with any Convergence Test(s).

Ex: $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$ Related Function $f(x) = \dfrac{\ln x}{x}$ continuous $(x > 0)$, positive $(x > 1)$,

decreasing $f'(x) = \dfrac{1 - \ln x}{x^2} < 0$ for $x > e$. Study the Related Integral

$$\int_2^{\infty} \frac{\ln x}{x}\,dx = \lim_{t\to\infty} \int_2^{t} \frac{\ln x}{x}\,dx = \lim_{t\to\infty} \int_{\ln 2}^{\ln t} u\,du = \lim_{t\to\infty} \frac{u^2}{2}\bigg|_{\ln 2}^{\ln t} = \lim_{t\to\infty} \frac{(\cancel{\ln t})^{2}}{2} - \frac{(\ln 2)^2}{2} = \boxed{\infty}$$

$\boxed{\begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array}}$ $\boxed{\begin{array}{l} x = 2 \Rightarrow u = \ln 2 \\ x = t \Rightarrow u = \ln t \end{array}}$

Therefore, the Improper Integal Diverges. As a result, the Original Series also **Diverges by the Integral Test**.

Ex: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+7} \approx \sum_{n=1}^{\infty} \frac{1}{n^3}$ $\leftarrow$ Comparison Series: Convergent p-series $p = 3 > 1$

Bound Terms

$\dfrac{1}{n^3+7} \leq \dfrac{1}{n^3}$. Therefore, the Original Series also **Converges by the Comparison Test.**

Ex: $\displaystyle\sum_{n=1}^{\infty} \frac{n^3+2}{n^4+8} \approx \sum_{n=1}^{\infty} \frac{n^3}{n^4} = \sum_{n=1}^{\infty} \frac{1}{n}$ $\leftarrow$ Comparison Series: Divergent p-series $p = 1$

Next check: $\displaystyle\lim_{n\to\infty} \frac{\dfrac{n^3+2}{n^4+8}}{\dfrac{1}{n}} = \lim_{n\to\infty} \frac{n^4+2n}{n^4+8}\frac{\left(\frac{1}{n^4}\right)}{\left(\frac{1}{n^4}\right)} = \lim_{n\to\infty} \frac{1+\cancelto{0}{\frac{2}{n^3}}}{1+\cancelto{0}{\frac{8}{n^4}}} = 1$ Finite and Non-Zero

Therefore, the Original Series also **Diverges by the Limit Comparision Test**.

Continue to NEXT Page for HW problems.

Use the Integral Test to determine whether the given series Converges or Diverges. You do **NOT** need to check the 3 pre-conditions for the Integral Test this time.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ 2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ 3. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ 4. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$

5. Consider $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4}$. Use **two** Different methods, namely the Integral Test (no pre-Condition check needed) and the Comparison Test, to prove that this series Converges.

Determine if the series Converges or Diverges using either the Comparison **OR** Limit Comparison Test.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$ 7. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+5}{n^3}$ 8. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}+2}$ 9. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+7}{n^7+2}$

10. Consider $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2+n}{n^4}$. Use **two** Different methods to prove that this series Converges. Use the Limit Comparison Test and then a *split-split* algebra technique into p-series pieces.

Determine whether the given series Converges or Diverges. Justify.

11. $\displaystyle\sum_{n=1}^{\infty} \sin^2\left(\frac{\pi n^4+1}{6n^4+5}\right)$ 12. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(\pi n^4+1)}{6n^4+5}$ 13. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{n^9}+\frac{7^n}{9^n}$

REVIEW

14. $\displaystyle\sum_{n=1}^{\infty} n^6+6$ 15. $\displaystyle\sum_{n=1}^{\infty} \frac{n^6+6}{n^6+1}$ 16. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^6+1}$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Gretta, SMUDD 208

Tuesday: 1:00–4:00 pm

7:30–9:00 pm TA Aidee, SMUDD 208

9–10:30 pm TA Natalie, SMUDD 208

Wednesday: 1:00-3:00 pm

7:30–9:00 pm TA Gretta, SMUDD 208

Thursday: none for Professor

7:30–9:00 pm TA Aidee, SMUDD 208

9:00–10:30 pm TA Natalie, SMUDD 208

Friday: 12:00–2:00 pm

Train your Convergence Tests Daily