Math 121, Section(s) 01, Spring 2024

Homework #10

Due **Wednesday, March 6th** in Gradescope by 11:59 pm ET

Goal: Exploring Limits of Infinite Sequences. We may also need L'Hôpital's Rule to finish some of the limits at hand.

FIRST: Read through and understand the following 5 Examples.

Determine whether the given sequence Converges or Diverges. If it converges, find the Limit.

Ex: $\left\{\dfrac{\ln n}{n^3}\right\}_{n=1}^{\infty} \hookrightarrow \lim\limits_{n\to\infty}\dfrac{\ln n}{n^3} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to\infty}\dfrac{\ln x}{x^3} \overset{\text{L'H}}{=} \lim\limits_{x\to\infty}\dfrac{\frac{1}{x}}{3x^2} = \lim\limits_{x\to\infty}\dfrac{1}{3x^3} \overset{\frac{1}{\infty}}{=} \boxed{0}$ Converges

Ex: $\left\{\dfrac{e^n}{n^2}\right\}_{n=1}^{\infty} \hookrightarrow \lim\limits_{n\to\infty}\dfrac{e^n}{n^2} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to\infty}\dfrac{e^x}{x^2} \overset{\text{L'H}}{=} \lim\limits_{x\to\infty}\dfrac{e^x}{2x} = \lim\limits_{x\to\infty}\dfrac{e^x}{2} = \boxed{\infty}$ Diverges

Ex: $\left\{\dfrac{4-9n^3}{5n^3+8n^2-7n-6}\right\}_{n=1}^{\infty}$ Note: Can switch to the Related Function in x and use L'H Rule or ... Quicker ...

$\hookrightarrow \lim\limits_{n\to\infty}\dfrac{4-9n^3}{5n^3+8n^2-7n-6}\dfrac{\left(\frac{1}{n^3}\right)}{\left(\frac{1}{n^3}\right)} = \lim\limits_{n\to\infty}\dfrac{\frac{4}{n^3}-9}{5+\frac{8}{n}-\frac{7}{n^2}-\frac{6}{n^3}} \overset{\frac{0}{0}}{=} \boxed{-\dfrac{9}{5}}$ Converges

Ex: $\left\{\left(1-\sin\left(\dfrac{6}{n^3}\right)\right)^{n^3}\right\}_{n=1}^{\infty} \hookrightarrow \lim\limits_{n\to\infty}\left(1-\sin\left(\dfrac{6}{n^3}\right)\right)^{n^3} \overset{1^{\infty}}{=} \lim\limits_{x\to\infty}\left(1-\sin\left(\dfrac{6}{x^3}\right)\right)^{x^3}$

$= e^{\lim\limits_{x\to\infty}\ln\left[\left(1-\sin\left(\frac{6}{x^3}\right)\right)^{x^3}\right]} = e^{\lim\limits_{x\to\infty} x^3\ln\left(1-\sin\left(\frac{6}{x^3}\right)\right)} \overset{\infty\cdot 0}{=} e^{\lim\limits_{x\to\infty}\frac{\ln\left(1-\sin\left(\frac{6}{x^3}\right)\right)^{\frac{0}{0}}}{\frac{1}{x^3}}}$

$\overset{\text{L'H}}{=} e^{\lim\limits_{x\to\infty}\frac{\frac{1}{1-\sin\left(\frac{6}{x^3}\right)}\cdot\left(-\cos\left(\frac{6}{x^3}\right)\right)^{-1}\cdot\left(-\frac{18}{x^4}\right)^{6}}{-\frac{3}{x^4}}} = e^{-6} = \boxed{\dfrac{1}{e^6}}$ Converges

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Ex:

$$\left\{\frac{(3n-1)!}{(3n+1)!}\right\}_{n=1}^{\infty}$$

$$\hookrightarrow \lim_{n\to\infty}\frac{(3n-1)!}{(3n+1)!} = \lim_{n\to\infty}\frac{\cancel{(3n-1)!}}{(3n+1)(3n)\cancel{(3n-1)!}} = \lim_{n\to\infty}\frac{1}{(3n+1)(3n)} = \boxed{0}$$ Converges

Next do the following HW problems.

List the first five terms of the Sequence. (Start with $n = 1$)

1. $a_n = \dfrac{(-1)^{n-1}}{5^n}$

2. $a_n = \dfrac{1}{(n+1)!}$

3. $a_n = \dfrac{(-1)^n\, n^2}{n+1}$

Determine whether the given sequence Converges or Diverges. If it converges, find the Limit. Justify, no guessing here.

4. $\left\{\dfrac{n}{n+1}\right\}_{n=1}^{\infty}$

5. $\left\{\dfrac{5n^2+3}{2n^2-7n}\right\}_{n=1}^{\infty}$

6. $\left\{\dfrac{3n^4-n-5}{7n^4+n^2-9}\right\}_{n=1}^{\infty}$

7. $\left\{\dfrac{\tan^{-1} n}{n}\right\}$

8. $\left\{\dfrac{n^2}{e^n}\right\}$

9. $\left\{n\sin\left(\dfrac{1}{n}\right)\right\}$

10. $\left\{\dfrac{(\ln n)^2}{n}\right\}_{n=1}^{\infty}$

11. $\left\{\dfrac{n^{99}}{\ln n}\right\}_{n=2}^{\infty}$

12. $\left\{\dfrac{\ln(99)}{n^{99}}\right\}$

13. $\left\{\left(1+\dfrac{1}{n}\right)^n\right\}_{n=1}^{\infty}$

14. $\left\{\left(1-\dfrac{5}{n^6}\right)^{n^6}\right\}_{n=1}^{\infty}$

15. $\left\{\left(1-\arcsin\left(\dfrac{3}{n^2}\right)\right)^{n^2}\right\}$

16. $\{\ln(2n^2+1)-\ln(n^2+1)\}$

17. $\left\{\dfrac{(n+3)!}{(n+1)!}\right\}_{n=1}^{\infty}$

18. $\left\{\dfrac{(2n-1)!}{(2n+1)!}\right\}$

19. $\left\{\cos^2\left(\dfrac{\pi n^6+6}{6n^6+1}\right)\right\}_{n=1}^{\infty}$

20. $\left\{\arctan\left(\dfrac{5n^7+1}{5n^7+7}\right)\right\}_{n=1}^{\infty}$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Gretta, SMUDD 208

Tuesday: 1:00–4:00 pm

7:30–9:00 pm TA Aidee, SMUDD 208

9–10:30 pm TA Natalie, SMUDD 208

Wednesday: 1:00-3:00 pm

7:30–9:00 pm TA Gretta, SMUDD 208

Thursday: none for Professor

7:30–9:00 pm TA Aidee, SMUDD 208

9:00–10:30 pm TA Natalie, SMUDD 208

Friday: 12:00–2:00 pm

dig deep, check notation, reference, justify, search, clarify...
challenge to everyone this week, get help on a challenging problem
in office hours with me or a Math Fellow