Math 121, Section(s) 01, Spring 2024

Homework #1

Due **Wednesday, January 31st** in Gradescope by 11:59 pm ET

Goal: Review of Integration using u-substitution:

FIRST: Read through and understand the following eight Examples.

NOTE: For now, show all of your work. In the future, when we are more experienced, we may back off on showing all steps for u-substitution, but not yet.

Compute the following Integrals.

INDEFINITE Integrals: Always remember to add $+C$ right away, as soon as you compute the *Most General Antiderivative.*

1. $\displaystyle\int e^x \cos(e^x)\ dx = \int \cos u\ du = \sin u + C = \boxed{\sin(e^x) + C}$

Here $\boxed{\begin{aligned} u &= e^x \\ du &= e^x dx \end{aligned}}$

2. $\displaystyle\int \frac{\cos x}{e^{\sin x}}\ dx = \int \frac{1}{e^u}\ du = \int e^{-u}\ du \overset{k\text{-rule}}{=} -e^{-u} + C = \boxed{-e^{-\sin x} + C} \overset{\text{or}}{=} \boxed{-\frac{1}{e^{\sin x}} + C}$

Here $\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x dx \end{aligned}}$ Recall k-rule: $\boxed{\displaystyle\int e^{kx}\ dx = \frac{1}{k}e^{kx} + C}$ for some constant $k \neq 0$

3. $\displaystyle\int e^{2x}\left(4 - e^{2x}\right)^6\ dx = -\frac{1}{2}\int u^6\ du = -\frac{1}{2}\cdot\frac{u^7}{7} + C = \boxed{-\frac{(4-e^{2x})^7}{14} + C}$

Here $\boxed{\begin{aligned} u &= 4 - e^{2x} \\ du &= -2e^{2x}dx \\ -\tfrac{1}{2}du &= e^{2x}dx \end{aligned}}$

4. $\displaystyle\int \frac{7}{e^{4x}\left(6 + e^{-4x}\right)}\ dx = -\frac{7}{4}\int \frac{1}{u}\ du = -\frac{7}{4}\ln|u| + C = \boxed{-\frac{7}{4}\ln\left|6 + e^{-4x}\right| + C}$

Here $\boxed{\begin{aligned} u &= 6 + e^{-4x} \\ du &= -4e^{-4x}dx \\ -\tfrac{1}{4}du &= e^{-4x}dx \\ -\tfrac{1}{4}du &= \frac{1}{e^{4x}}dx \end{aligned}}$ Note: Can drop Absolute Value, because $6 + e^{-4x}$ is postive.

DEFINITE Integrals: Recall, you must *change* or *mark* your Limits of integration.

5. $\displaystyle\int_{\ln 2}^{\ln 6} \frac{e^x}{2+e^x}\,dx = \int_4^8 \frac{1}{u}\,du = \ln|u|\Big|_4^8 = \ln 8 - \ln 4 = \ln\left(\frac{8}{4}\right) = \boxed{\ln 2}$

Here $\boxed{\begin{aligned} u &= 2+e^x \\ du &= e^x\,dx \end{aligned}}$ and $\boxed{\begin{aligned} x = \ln 2 &\implies u = 2+e^{\ln 2} = 2+2 = 4 \\ x = \ln 6 &\implies u = 2+e^{\ln 6} = 2+6 = 8 \end{aligned}}$

6. $$\begin{aligned}\int_e^{e^3} \frac{4}{x(\ln x)^2}\,dx &= 4\int_1^3 \frac{1}{u^2}\,du = 4\int_1^3 u^{-2}\,du = -4u^{-1}\Big|_1^3 = -\frac{4}{u}\Big|_1^3 = -\frac{4}{3} - (-4) \\ &= -\frac{4}{3} + 4 = -\frac{4}{3} + \frac{12}{3} = \boxed{\frac{8}{3}}\end{aligned}$$

Here $\boxed{\begin{aligned} u &= \ln x \\ du &= \tfrac{1}{x}\,dx \end{aligned}}$ and $\boxed{\begin{aligned} x = e &\implies u = \ln e = 1 \\ x = e^3 &\implies u = \ln e^3 = 3 \end{aligned}}$

7. Here is an example of an *inverted* or *reverse* substitution.

$$\begin{aligned}\int x\sqrt{x+1}\,dx &= \int (u-1)\sqrt{u}\,du = \int u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du = \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + C \\ &= \boxed{\tfrac{2}{5}(x+1)^{\frac{5}{2}} - \tfrac{2}{3}(x+1)^{\frac{3}{2}} + C}\end{aligned}$$

Here $\boxed{\begin{aligned} u &= x+1 \Rightarrow x = u-1 \\ du &= dx \end{aligned}}$

8. Here is an example if you prefer to *mark* your limits of integration for u-substitution, instead of changing them. Generally, we will *change* the limits in class.

$$\begin{aligned}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan x\,dx &= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\cos x}\,dx = -\int_{x=\frac{\pi}{6}}^{x=\frac{\pi}{3}} \frac{1}{u}\,du = -\ln|u|\Big|_{x=\frac{\pi}{6}}^{x=\frac{\pi}{3}} = -\ln|\cos x|\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} \\ &= -\left(\ln\left|\cos\left(\frac{\pi}{3}\right)\right| - \ln\left|\cos\left(\frac{\pi}{6}\right)\right|\right) = -\left(\ln\left(\frac{1}{2}\right) - \ln\left(\frac{\sqrt{3}}{2}\right)\right) \\ &= -\left((\cancelto{0}{\ln 1} - \cancel{\ln 2}) - (\ln\sqrt{3} - \cancel{\ln 2})\right) = -\left(-\ln\sqrt{3}\right) = \boxed{\ln\sqrt{3}} \overset{\text{or}}{=} \boxed{\frac{\ln 3}{2}}\end{aligned}$$

Here $\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x dx \\ -du &= \sin x dx \end{aligned}}$

Next, complete the following HW problems.

Please try to work these integrals quickly, yet still fully justify your solution. Again, we will likely guess u-subs in the future, but not yet. See Section 4.5 (Stewart Calculus) or the Integration Review video if needed. Plenty of help in Office Hours!

1. $\displaystyle\int \frac{1}{e^{7x}}\, dx$

2. $\displaystyle\int e^{14x}\, dx$

3. $\displaystyle\int e^{1-2x}\, dx$

4. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

5. $\displaystyle\int e^{x}\sin\left(e^{x}\right)\, dx$

6. $\displaystyle\int \cos\left(\frac{x}{5}\right)\, dx$

7. $\displaystyle\int \sec^2\theta\tan^3\theta\, d\theta$

8. $\displaystyle\int (2-3x)^5\, dx$

9. $\displaystyle\int \frac{1}{7-x}\, dx$

10. $\displaystyle\int \frac{1}{\sqrt{7x+5}}\, dx$

11. $\displaystyle\int \frac{1}{(3-5x)^2}\, dx$

12. $\displaystyle\int \frac{1}{2x-1}\, dx$

13. $\displaystyle\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\, dx$

14. $\displaystyle\int x\sqrt{7-3x^2}\, dx$

15. $\displaystyle\int \frac{1}{x\ln x}\, dx$

16. $\displaystyle\int \sin(\pi x+1)\, dx$

17. $\displaystyle\int x(1-x)^{79}\, dx$

18. $\displaystyle\int x^3(x+1)^{79}\, dx$

19. $\displaystyle\int \frac{x^2}{\sqrt{3-x}} dx$

20. $\displaystyle\int_0^{\ln 2} \frac{1}{e^{3x}\left(2-e^{-3x}\right)^2} dx$ mark or change limits of int.

you will likely need help on 17-20, so jump into Office Hours

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

Tuesday: 1:00–4:00 pm

Wednesday: 1:00-3:00 pm

Friday: 12:00–2:00 pm

Math Fellow evening TA Help Hours TBD soon

• Please take the chance to stop by my office hours (unannounced) to meet me and ask any questions you have. If you can't make these hours, go ahead and make an appointment at a different time. I will try to accomodate everyone. Just e-mail me.

• Please turn in your homework by the deadline. Gradescope will LOCK YOU OUT at 11:59 pm.

• Please **TAG** your HW solution numbers in Gradescope. Ask for help if needed

• Write a **final** draft neatly in either pen or pencil or on a tablet. No mess. No Scratch-outs. Make sure that your scan is clear for all problems.

• You are responsible for writing up your own solutions, in your own words. Please read the Statement of Intellectual Responsibility from our class syllabus. **No** online solution sources. I will give zero credit for all work copied from any other source. I will also report you to the Dean of Conduct/Community Standards. You will also risk immediate Failure in the course.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.