



Math 121 Exam 3

April 28, 2023



- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\arctan \sqrt{3}$ or $\cosh(\ln 3)$ should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [16 Points] Find the **Interval** and **Radius** of Convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n (5x+1)^n}{(5n+1) \cdot 4^n}$

Analyze carefully and with full justification.

2. [22 Points] Find the MacLaurin Series for each of the functions. Also **STATE** the Radius of Convergence for each series. Answers should be in sigma notation $\sum_{n=0}^{\infty}$. Simplify.

(a) $\ln\left(1 + \frac{x^2}{4}\right)$

(b) $6x^3 \arctan(6x)$

(c) $\frac{d}{dx} (8x^4 \sin(8x))$

(d) $\int x^3 e^{-x^4} dx$

3. [16 Points] (a) Use Series to Estimate $\int_0^1 x^2 \cos(x^3) dx$ with error less than $\frac{1}{50}$.

Hint: $(24) \cdot (15) = 360$

(b) Estimate $\frac{1}{e}$ with error less than $\frac{1}{100}$

4. [24 Points] Find the **sum** for each of the following convergent series. Simplify, if possible.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{9^n (2n+1)!}$ (b) $-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \dots$ (c) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} \pi^{2n+1}}{4! (2n)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} (\ln 3)^n}{5 \cdot n!}$ (e) $-\frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \dots$ (f) $\pi^2 - \frac{\pi^4}{3!} + \frac{\pi^6}{5!} - \frac{\pi^8}{7!} + \dots$

5. [10 Points] Use Series to compute the following Limit $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{e^{-x} - 1 + x}$

6. [12 Points] Find the MacLaurin Series Representation for $\ln(9 + x^2)$.

Hint: $\ln(9 + x^2) = \int \frac{2x}{9 + x^2} dx$. Yes, solve for C

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute $\sum_{n=0}^{\infty} \frac{n}{3^n}$

OPTIONAL BONUS #2

Determine the 27th and 28th derivatives for $f(x) = x^3 \arctan(x^5)$ evaluated at $x = 0$. You do *not* need to simplify your answers here.