Math 121 Midterm Exam #3 May 9, 2022

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\arctan\sqrt{3}$ or $\cosh(\ln 3)$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [16 Points] Find the **Interval** and **Radius** of Convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (6x+1)^n}{(6n+1)\cdot 7^n}$

Analyze carefully and with full justification.

2. [12 Points] Find the MacLaurin Series for each of the following functions. **State** the Radius of Convergence for each series. Your answers should all be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$ here. Simplify.

(a) $\dfrac{x^2}{4+x} = x^2\left(\dfrac{1}{4+x}\right)$

(b) $8x^4 \arctan(8x)$

3. [16 Points] Your answers should all be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$ here. Simplify.

(a) Use Series to compute $\dfrac{d}{dx}\left(5x^2 e^{-x^3}\right)$.

(b) Use Series to compute $\displaystyle\int x^3 \sin\left(8x^4\right)\ dx$.

4. [10 Points] Use the Series to **Estimate** $\dfrac{1}{e}$ with error less than $\dfrac{1}{20}$. Justify.

5. [26 Points] Find the **sum** for each of the following convergent series. Simplify, if possible.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \ \pi^{2n}}{(16)^n \ (2n+1)!}$

(b) $3 + 3 - \frac{3}{2} + 1 - \frac{3}{4} + \frac{3}{5} - \frac{3}{6} + \frac{3}{7} - \ldots$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1} \ \pi^{2n+1}}{4! \ (2n)!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \ (\ln 8)^n}{3^{n+1} \ n!}$

(e) $-\frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \frac{\pi^9}{9!} - \ldots$

(f) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^n}$ Hint: $3 = \left(\sqrt{3}\right)^2$

6. [10 Points] Use Series and Integration to Derive the following MacLaurin Series formula:

$$\ln(1+6x) = \sum_{n=0}^{\infty} \frac{(-1)^n \ 6^{n+1} \ x^{n+1}}{n+1}$$

USE the following helpful formula: $\ln(1+6x) = \displaystyle\int \frac{6}{1+6x} \, dx$

Yes, show that $C = 0$. Answer should be in Sigma notation $\displaystyle\sum_{n=0}^{\infty}$

7. [10 Points]

Consider the Parametric Curve given by $x = (\arctan t) - t$ and $y = 2\sinh^{-1} t$.

Compute the Arclength of this parametric curve for $0 \le t \le \sqrt{3}$.

Hint: $\displaystyle\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{1+x^2}}$