



# Math 121 Exam 1

## February 23, 2024



- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- Numerical answers such as  $\sin\left(\frac{\pi}{6}\right)$ ,  $4^{\frac{3}{2}}$ ,  $\arctan(\sqrt{3})$ ,  $e^{\ln 4}$ ,  $\ln(e^7)$ , or  $e^{3\ln 3}$  should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

**1. Limits** [32 Points total] Evaluate each of the following. Please justify/simplify.

(a) Show that  $\lim_{x \rightarrow 0} \frac{\ln(1+5x) - 5x}{\arcsin(3x) + e^{-3x} - 1} = \boxed{-\frac{25}{9}}$

(b) Show that  $\lim_{x \rightarrow 0^+} x^3 \ln x = \boxed{0}$

(c) Show that  $\lim_{x \rightarrow \infty} \left(1 - \arctan\left(\frac{3}{x^4}\right)\right)^{x^4} = \boxed{e^{-3}}$

**Integrals** [36 Points total] Compute each of the following Definite Integrals. Justify.

**2.** Show that  $\int_{-3}^3 \sqrt{9-x^2} dx = \boxed{\frac{9\pi}{2}}$

**3.** Show that  $\int_0^{\ln \sqrt{3}} \frac{e^{2x}}{3+e^{4x}} dx = \boxed{\frac{\pi}{12\sqrt{3}}}$

**4.** Show that  $\int_1^e x^3 \ln x dx = \boxed{\frac{1+3e^4}{16}}$

**More Integrals** [32 Points total] Compute each of the following Indefinite Integrals. Justify.

**5.** Compute  $\int x^2 \arcsin x dx$

**6.** Compute  $\int \frac{x}{(4+x^2)^{\frac{7}{2}}} dx$  using a Trig Substitution Hint:  $\tan \theta = \frac{\sin \theta}{\cos \theta}$