Geometric Series Test

Consider a Geometric series $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \dots \begin{cases} \text{Converges if } |r| < 1, \\ \quad \text{with SUM} = \dfrac{a}{1-r} \\ \text{Diverges if } |r| \geq 1 \end{cases}$

n^{th} Term Divergence Test

Consider any series $\sum_{n=1}^{\infty} a_n$. If $\lim_{n\to\infty} a_n \neq 0$, then the Series Diverges

Integral Test

Consider a series $\sum_{n=1}^{\infty} a_n$ where the terms $a_n = f(n)$ and the related function $f(x)$ is continuous, positive, and decreasing on $[1, \infty)$

1. If the $\int_1^{\infty} f(x)\, dx$ Converges, then the series $\sum_{n=1}^{\infty} a_n$ Converges
2. If the $\int_1^{\infty} f(x)\, dx$ Diverges, then the series $\sum_{n=1}^{\infty} a_n$ Diverges

p-Series Test

The p-series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \dots \begin{cases} \text{Converges if } p > 1 \\ \text{Diverges if } p \leq 1 \end{cases}$

Comparison Test

Consider two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ with positive terms.

1. If $a_n \leq b_n$ and if $\sum_{n=1}^{\infty} b_n$ Converges, then $\sum_{n=1}^{\infty} a_n$ Converges.
2. If $a_n \geq b_n$ and if $\sum_{n=1}^{\infty} b_n$ Diverges, then $\sum_{n=1}^{\infty} a_n$ Diverges.

Limit Comparison Test

$$\text{Consider two series } \sum_{n=1}^{\infty} a_n \text{ and } \sum_{n=1}^{\infty} b_n \text{ with positive terms.}$$

$$\text{Suppose that } \lim_{n\to\infty} \frac{a_n}{b_n} = C \text{ with } 0 < C < \infty. \text{ Then}$$

1. If $\sum_{n=1}^{\infty} b_n$ Converges, then $\sum_{n=1}^{\infty} a_n$ Converges.
2. If $\sum_{n=1}^{\infty} b_n$ Diverges, then $\sum_{n=1}^{\infty} a_n$ Diverges.

Alternating Series Test

$$\text{Consider an Alternating series } \sum_{n=1}^{\infty} (-1)^{n+1}\, b_n = b_1 - b_2 + b_3 - b_4 + b_5 - \dots$$

Then if the following three conditions are all satisfied:

$$\left\{\begin{array}{l} \text{1. Isolate } b_n > 0 \\ \text{2. } \lim_{n\to\infty} b_n = 0 \\ \text{3. Terms Decreasing: } b_{n+1} \le b_n \end{array}\right\} \text{ then the Alternating series } \sum_{n=1}^{\infty} (-1)^{n+1}\, b_n \text{ Converges.}$$

Absolute Convergence Test

$$\text{Given a series } \sum_{n=1}^{\infty} a_n,$$

$$\text{if the Absolute Series } \sum_{n=1}^{\infty} |a_n| \text{ Converges, then the Original Series } \sum_{n=1}^{\infty} a_n \text{ Converges.}$$

Ratio Test

$$\text{Given a series } \sum_{n=1}^{\infty} a_n. \quad \text{Consider } L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|, \text{ if}$$

1. $L < 1$ then the Original Series is **Absolutely** Convergent
2. $L > 1$ then the Original Series Diverges
3. $L = 1$ INCONCLUSIVE