Math 121 Self-Assessment Quiz #3

Answer Key

- Please see the course webpage for the answer key.

1. Compute $\lim_{x\to\infty}\left(\frac{6}{x}\right)^{\frac{1}{2+\ln x}}$ $(0^0) = \lim_{x\to\infty} e^{\ln\left(\left(\frac{6}{x}\right)^{\frac{1}{2+\ln x}}\right)} = e^{\lim_{x\to\infty}\ln\left(\left(\frac{6}{x}\right)^{\frac{1}{2+\ln x}}\right)}$

$$= e^{\lim_{x\to\infty}\frac{\ln\left(\frac{6}{x}\right)}{2+\ln x}\left(\frac{-\infty}{\infty}\right)} \overset{\text{L'H}}{=} e^{\lim_{x\to\infty}\frac{\frac{1}{\left(\frac{6}{x}\right)}\left(-\frac{6}{x^2}\right)}{\frac{1}{x}}} = e^{\lim_{x\to\infty}\frac{\left(\frac{x}{6}\right)\left(-\frac{6}{x^2}\right)}{\frac{1}{x}}}$$

$$= e^{\lim_{x\to\infty}\frac{-\frac{1}{x}}{\frac{1}{x}}} = \boxed{e^{-1}}$$

2. Compute $\lim_{x\to0}\frac{\cosh(4x)-1-\arctan(4x)+4x}{\ln(1-x)+\arcsin x}\left(\frac{0}{0}\right) \overset{\text{L'H}}{=} \lim_{x\to0}\frac{4\sinh(4x)-\frac{4}{1+16x^2}+4}{-\frac{1}{1-x}+\frac{1}{\sqrt{1-x^2}}}\left(\frac{0}{0}\right)$

$$\overset{\text{L'H}}{=} \lim_{x\to0}\frac{16\cosh(4x)+\frac{4(32x)}{(1+16x^2)^2}}{-\frac{1}{(1-x)^2}-\frac{1}{2(1-x^2)^{\frac{3}{2}}}(-2x)} = \frac{16+0}{-1+0} = \boxed{-16}$$

3. $\lim_{x\to0}\frac{1-e^{-3x}-\arctan(3x)}{x^2}\left(\frac{0}{0}\right) \overset{\text{L'H}}{=} \lim_{x\to0}\frac{3e^{-3x}-\frac{3}{1+9x^2}}{2x}\left(\frac{0}{0}\right)$

$$\overset{\text{L'H}}{=} \lim_{x\to0}\frac{-9e^{-3x}+\frac{3(18x)}{(1+9x^2)^2}}{2} = \boxed{-\frac{9}{2}}$$

4. $\lim_{x\to\infty}\left(1-\frac{2}{x^3}\right)^{7x^3} \overset{1^\infty}{=} \lim_{x\to\infty} e^{\ln\left(\left(1-\frac{2}{x^3}\right)^{7x^3}\right)} = \lim_{x\to\infty} e^{7x^3\ln\left(1-\frac{2}{x^3}\right)}$

$$\overset{\infty\cdot 0}{=} e^{\lim\limits_{x\to\infty} 7x^3 \ln\left(1-\frac{2}{x^3}\right)} = e^{\lim\limits_{x\to\infty} \frac{7\ln\left(1-\frac{2}{x^3}\right)}{\frac{1}{x^3}}} \overset{\left(\frac{0}{0}\right)^{\text{L'H}}}{=} e^{\lim\limits_{x\to\infty} \frac{7\left(\frac{1}{1-\frac{2}{x^3}}\right)\left(\frac{6}{x^4}\right)}{-\frac{3}{x^4}}}$$

$$= e^{\lim\limits_{x\to\infty} 7\left(\frac{1}{1-\frac{2}{x^3}}\right)\left(\frac{6}{x^4}\right)\left(-\frac{x^4}{3}\right)} = e^{\lim\limits_{x\to\infty} 7\left(\frac{1}{1-\frac{2}{x^3}}\right)(-2)} = \boxed{e^{-14}}$$

5. $$\lim_{x\to\infty}\left(\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}\right)^x = e^{\lim\limits_{x\to\infty} \ln\left(\left(\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}\right)^x\right)}$$

$$= e^{\lim\limits_{x\to\infty} x\ln\left(\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}\right)} = e^{\lim\limits_{x\to\infty} \frac{\ln\left(\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}\right)}{\frac{1}{x}}^{\frac{0}{0}}}$$

$$\overset{\text{L'H}}{=} e^{\lim\limits_{x\to\infty} \frac{\frac{1}{\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}}\left(\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\left(-\frac{1}{x^2}\right)+e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)\right)}{-\frac{1}{x^2}}}$$

$$= e^{\lim\limits_{x\to\infty} \frac{\frac{1}{\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}}\left(\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}+e^{\frac{1}{x}}\right)}{1}} = e^{1+1} = \boxed{e^2}$$

6. Compute

$$\int_0^{\sqrt{3}} x\arctan x\, dx \overset{(*)}{=} \frac{x^2}{2}\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} \frac{x^2}{1+x^2}\, dx$$

$$= \frac{x^2}{2}\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} \frac{x^2+1-1}{1+x^2}\, dx = \frac{x^2}{2}\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} \frac{x^2+1}{1+x^2} - \frac{1}{x^2+1}\, dx$$

$$= \frac{x^2}{2}\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} 1 - \frac{1}{1+x^2}\, dx = \frac{x^2}{2}\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}(x-\arctan x)\Big|_0^{\sqrt{3}}$$

$$= \frac{x^2}{2}\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}x + \frac{1}{2}\arctan x\Big|_0^{\sqrt{3}}$$

$$= \frac{1}{2}\left(3\arctan\sqrt{3} - 0\arctan 0\right) - \frac{1}{2}\sqrt{3} + 0 + \frac{1}{2}\arctan\sqrt{3} - \frac{1}{2}\arctan 0$$

$$= \frac{3}{2}\left(\frac{\pi}{3}\right) - 0 - \frac{\sqrt{3}}{2} + 0 + \frac{1}{2}\left(\frac{\pi}{3}\right) - 0 = \boxed{\frac{2\pi}{3} - \frac{\sqrt{3}}{2}}$$

(*) I.B.P.

$u = \arctan x$	$dv = x\,dx$
$du = \frac{1}{1+x^2}dx$	$v = \frac{x^2}{2}$

7. Show $\displaystyle\int_1^{e^4} \frac{\ln x}{\sqrt{x}}\,dx = 4e^2 + 4$

$$\int_1^{e^4} \frac{\ln x}{\sqrt{x}}\,dx = 2\sqrt{x}\ln x\Big|_1^{e^4} - 2\int_1^{e^4} \frac{1}{\sqrt{x}}\,dx$$

$$= 2\sqrt{x}\ln x\Big|_1^{e^4} - 4\sqrt{x}\Big|_1^{e^4} = 2\sqrt{e^4}\ln\left(e^4\right) - 2\ln 1 - \left[4\sqrt{e^4} - 4\sqrt{1}\right] = 8e^2 - 0 - 4e^2 + 4 = \boxed{4e^2 + 4}$$

(*) I.B.P.

$u = \ln x$	$dv = x^{-\frac{1}{2}}dx$
$du = \frac{1}{x}dx$	$v = 2\sqrt{x}$