Self-Assessment Quiz #11 Answers

1. $x = e^t + \dfrac{1}{1+e^t}$ $\qquad y = 2\ln(1+e^t)$ ← Does not Simplify

$\dfrac{dx}{dt} = e^t - \dfrac{e^t}{(1+e^t)^2}$ $\qquad \dfrac{dy}{dt} = \dfrac{2e^t}{1+e^t}$ ← chain rules here

(a) $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{2\cancel{e^t}}{1+e^t}}{\cancel{e^t} - \frac{\cancel{e^t}}{(1+e^t)^2}} = \frac{\frac{2}{1+e^t}}{1-\frac{1}{(1+e^t)^2}}$$ cancel e^t

$$\left.\frac{dy}{dx}\right|_{t=0} = \frac{\frac{2}{1+e^0}}{1-\frac{1}{(1+e^0)^2}} = \frac{\frac{2}{2}}{1-\frac{1}{(2)^2}} = \frac{1}{1-1/4} = \frac{1}{(3/4)} = \boxed{\frac{4}{3}}$$ ← slope @ $t=0$

Point: $x(0) = e^0 + \dfrac{1}{1+e^0} = 1 + \dfrac{1}{2} = 3/2$ $\qquad y(0) = 2\ln(1+e^0) = 2\ln 2$

Tangent Line

$$y - 2\ln 2 = 4/3\,(x - 3/2)$$

$$\boxed{y = 4/3\,x - 2 + 2\ln 2}$$

(b) $$L = \int_0^{\ln 3} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^{\ln 3} \sqrt{\left(e^t - \frac{e^t}{(1+e^t)^2}\right)^2 + \left(\frac{2e^t}{1+e^t}\right)^2}\,dt$$

$$= \int_0^{\ln 3} \sqrt{e^{2t} \underbrace{- \frac{2e^{2t}}{(1+e^t)^2} + \frac{e^{2t}}{(1+e^t)^4} + \frac{4e^{2t}}{(1+e^t)^2}}_{\text{combine}}}\,dt$$

1(b) Continued

$$= \int_0^{\ln 3} \sqrt{e^{2t} + \frac{2e^{2t}}{(1+e^t)^2} + \frac{e^{2t}}{(1+e^t)^4}}\, dt$$

$$= \int_0^{\ln 3} \sqrt{\left(e^t + \frac{e^t}{(1+e^t)^2}\right)^2}\, dt$$

$$= \int_0^{\ln 3} e^t + \frac{e^t}{(1+e^t)^2}\, dt$$

u-sub.?

$$= e^t - \frac{1}{1+e^t}\Big|_0^{\ln 3} = e^{\ln 3} - \frac{1}{1+e^{\ln 3}} - \left(e^0 - \frac{1}{1+e^0}\right)$$

$$= 3 - \frac{1}{4} - \frac{1}{2} = \frac{12}{4} - \frac{1}{4} - \frac{2}{4} = \boxed{\frac{9}{4}}$$

1(c)

$$S.A. = 2\pi \int_0^{\ln 3} y(t) \underbrace{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}}\, dt$$

from above (b), not the value for length but the $\sqrt{\text{root}}$ expression

$$\boxed{= 2\pi \int_0^{\ln 3} \left(e^t + \frac{1}{1+e^t}\right)\left(e^t + \frac{e^t}{(1+e^t)^2}\right) dt}$$

2. $x = t - e^{2t}$ $\qquad$ $y = 1 - \sqrt{8}\, e^{t}$

$\frac{dx}{dt} = 1 - 2e^{2t}$ $\qquad$ $\frac{dy}{dt} = -\sqrt{8}\, e^{t}$

$$S.A. = 2\pi \int_0^3 x(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 2\pi \int_0^3 (t - e^{2t}) \sqrt{(1-2e^{2t})^2 + (-\sqrt{8}e^{t})^2}\, dt$$

$$= 2\pi \int_0^3 (t - e^{2t}) \sqrt{1 \underbrace{- 4e^{2t} + 4e^{4t} + 8e^{2t}}_{\text{combine}}}\, dt = 2\pi \int_0^3 (t - e^{2t}) \sqrt{1 + 4e^{2t} + 4e^{4t}}\, dt$$

$$= 2\pi \int_0^3 (t - e^{2t}) \sqrt{(1 + 2e^{2t})^2}\, dt = 2\pi \int_0^3 \underbrace{(t - e^{2t})(1 + 2e^{2t})}_{\text{FOIL}}\, dt$$

$$= 2\pi \int_0^3 t + \underbrace{2te^{2t}}_{\text{IBP See}(*)} - e^{2t} - 2e^{4t}\, dt$$

$$= 2\pi \left[\frac{t^2}{2} + 2\left(\frac{te^{2t}}{2} - \frac{e^{2t}}{4} \right) - \frac{e^{2t}}{2} - \frac{2e^{4t}}{4_{\,2}} \right]_0^3$$

$$= 2\pi \left[\frac{t^2}{2} + te^{2t} \underbrace{- \frac{e^{2t}}{2} - \frac{e^{2t}}{2}}_{-e^{2t}} - \frac{e^{4t}}{2} \right]_0^3$$

$$= 2\pi \left[\left(\frac{9}{2} + 3e^{6} - e^{6} - \frac{e^{12}}{2} \right) - \left(0 + 0 - \cancel{e^{0}} - \frac{\cancel{e^{0}}}{2} \right) \right] = \boxed{2\pi \left[6 + 2e^{6} - \frac{e^{12}}{2} \right]}$$

$-\frac{3}{2}$

$\frac{9}{2} + \frac{3}{2} = \frac{12}{2}$

(*) $\int te^{2t}\, dt = \frac{te^{2t}}{2} - \frac{1}{2}\int e^{2t}\, dt = \boxed{\frac{te^{2t}}{2} - \frac{e^{2t}}{4} + C}$

$$\boxed{\begin{array}{ll} u = t & dv = e^{2t}dt \\ du = dt & v = \frac{e^{2t}}{2} \end{array}}$$