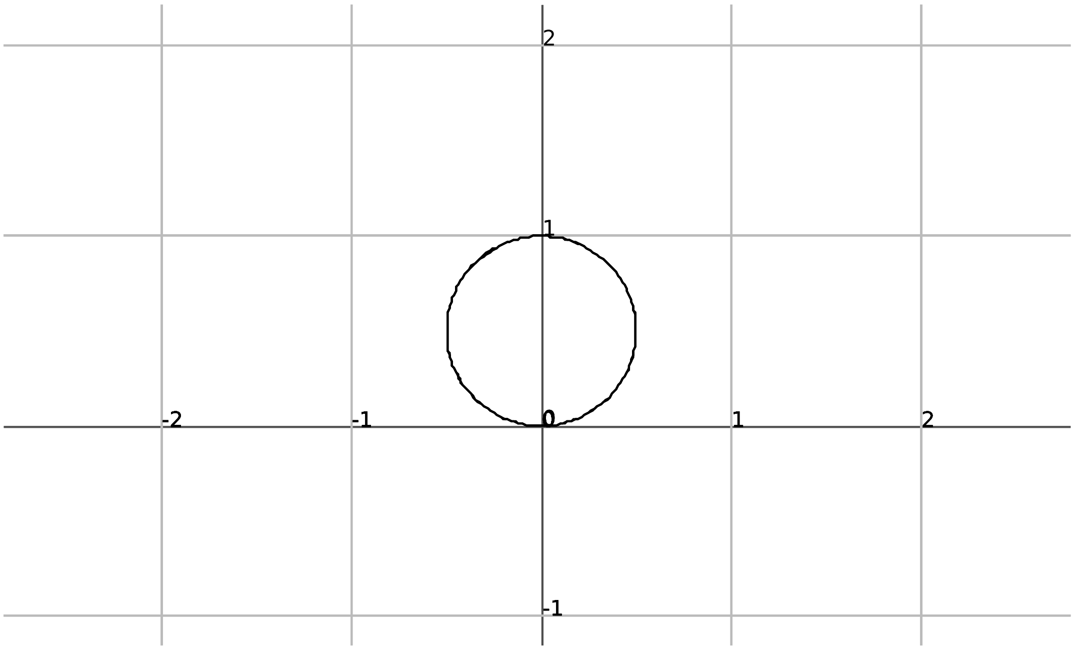
Basic Polar Curves Handout

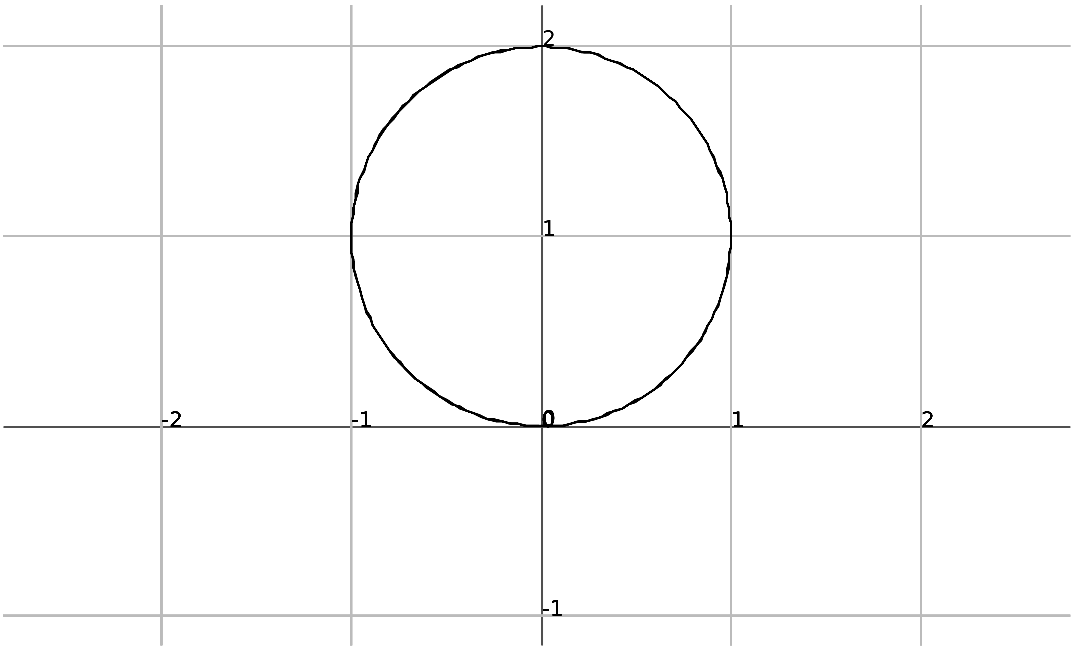
Math 121-D. Benedetto

Recognize a few of the following basic polar curves. Try and understand the sketches.

- $\boxed{r = \sin\theta}$ circle of radius $\frac{1}{2}$ centered at $\left(0, \frac{1}{2}\right)$

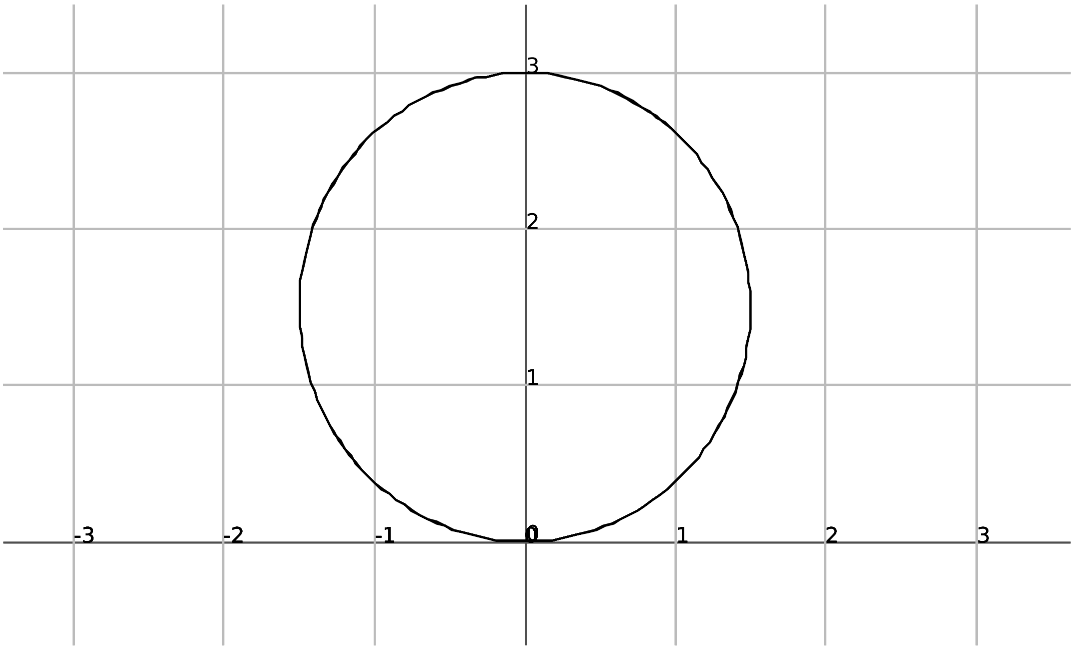


- $\boxed{r = 2\sin\theta}$ circle of radius 1 centered at $(0, 1)$



Note: These circles cycle through and close one full loop as θ ranges from say $\theta = 0$ to just $\theta = \pi$.

- $\boxed{r = 3\sin\theta}$ circle of radius $\frac{3}{2}$ centered at $(0, \frac{3}{2})$



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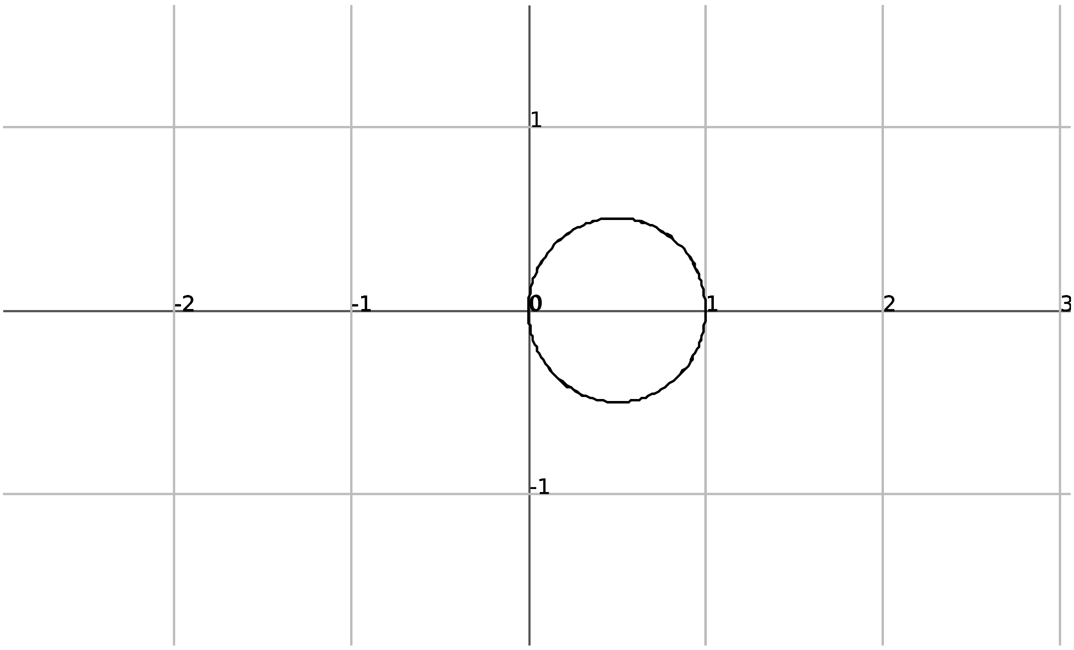
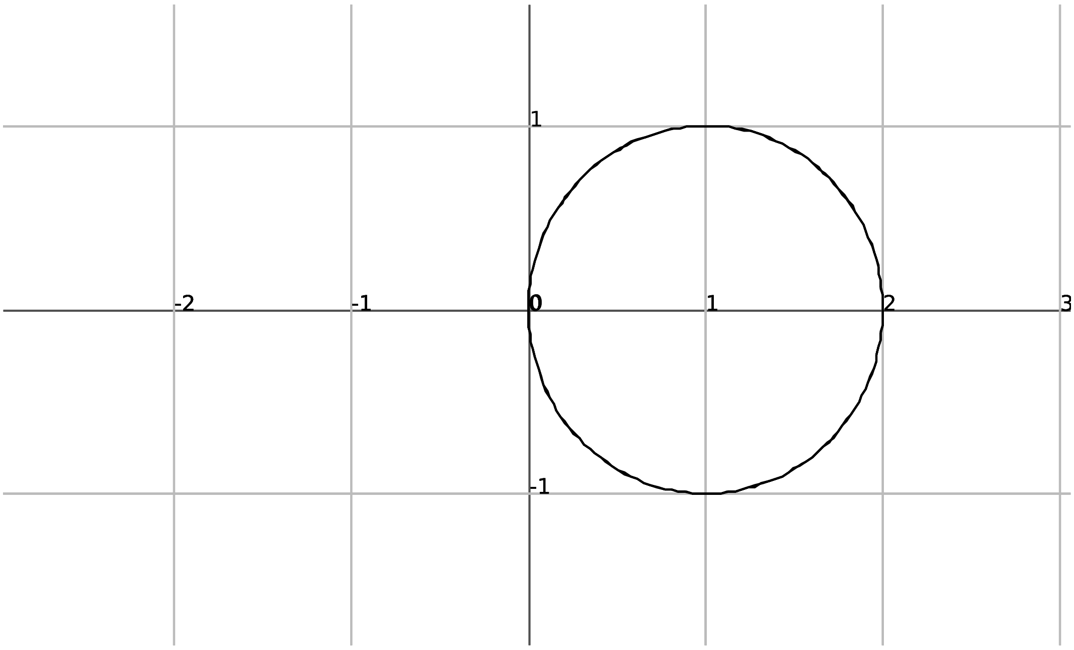
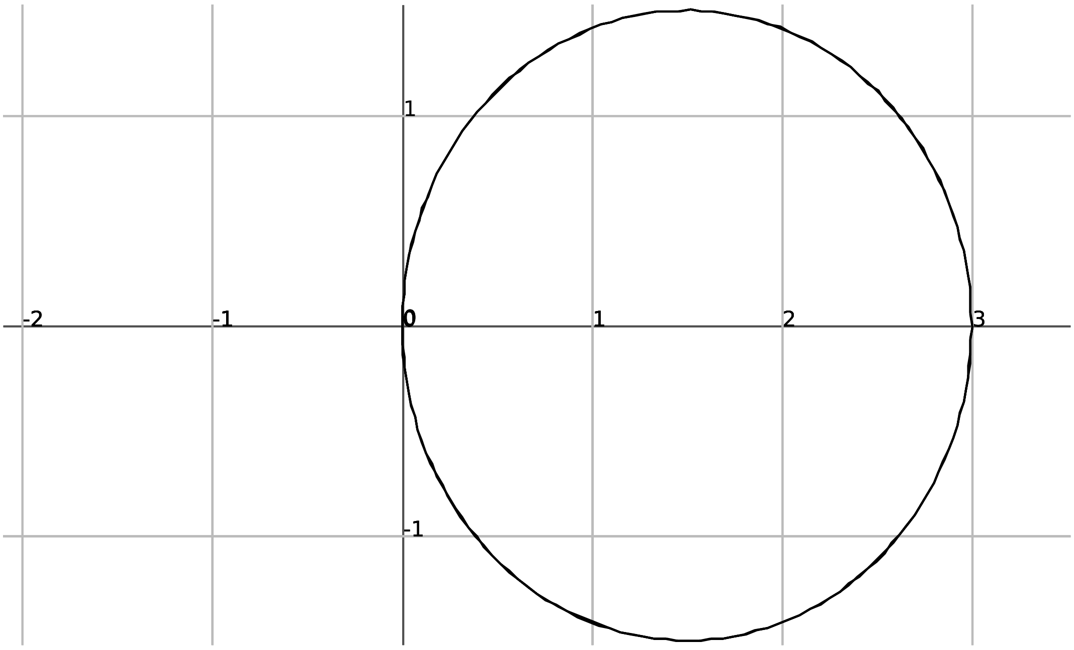
- $\boxed{r = \cos\theta}$ circle of radius $\frac{1}{2}$ centered at $\left(\frac{1}{2}, 0\right)$



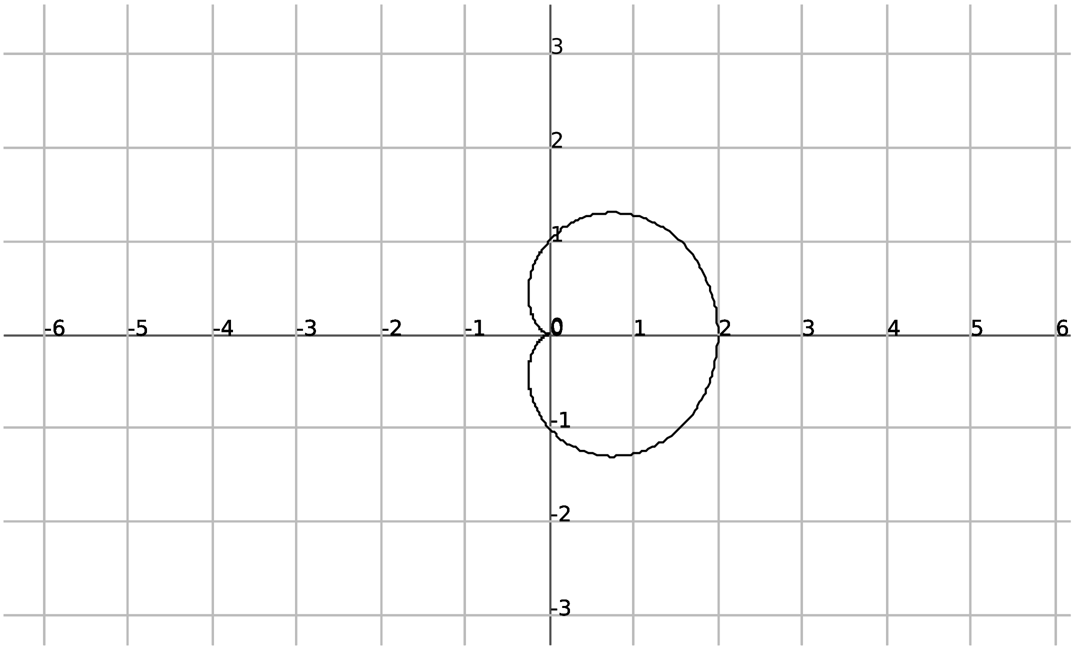
- $\boxed{r = 2\cos\theta}$ circle of radius 1 centered at $(1,0)$



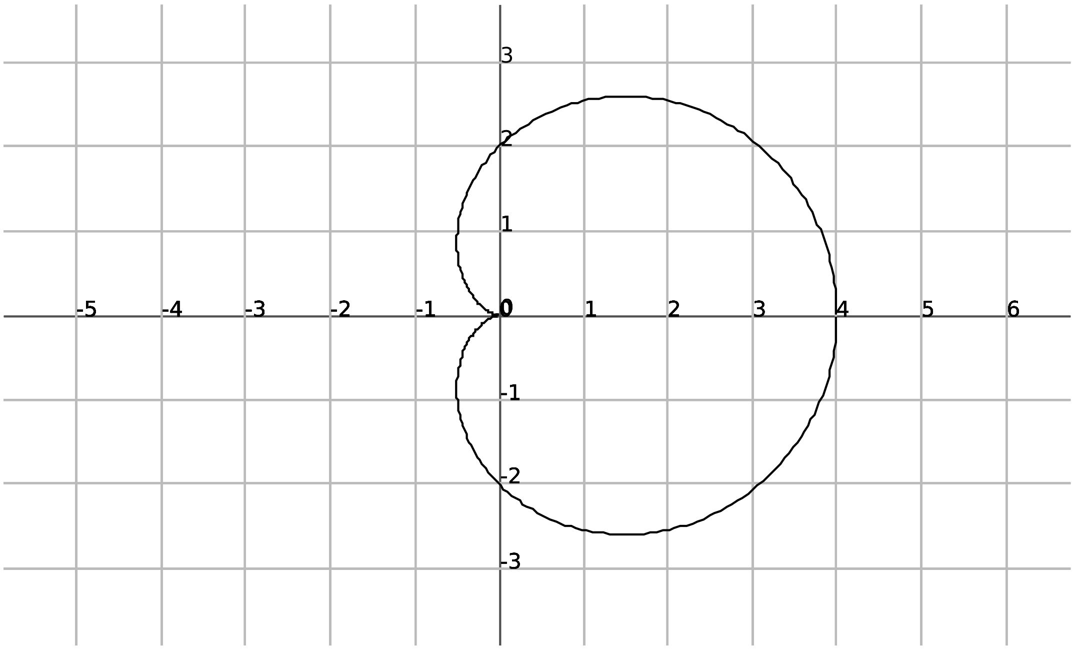
- $\boxed{r = 3\cos\theta}$ circle of radius $\frac{3}{2}$ centered at $(\frac{3}{2},0)$



- $\boxed{r = 1 + \cos\theta}$ cardioid (think about $r = 1 - \cos\theta$)

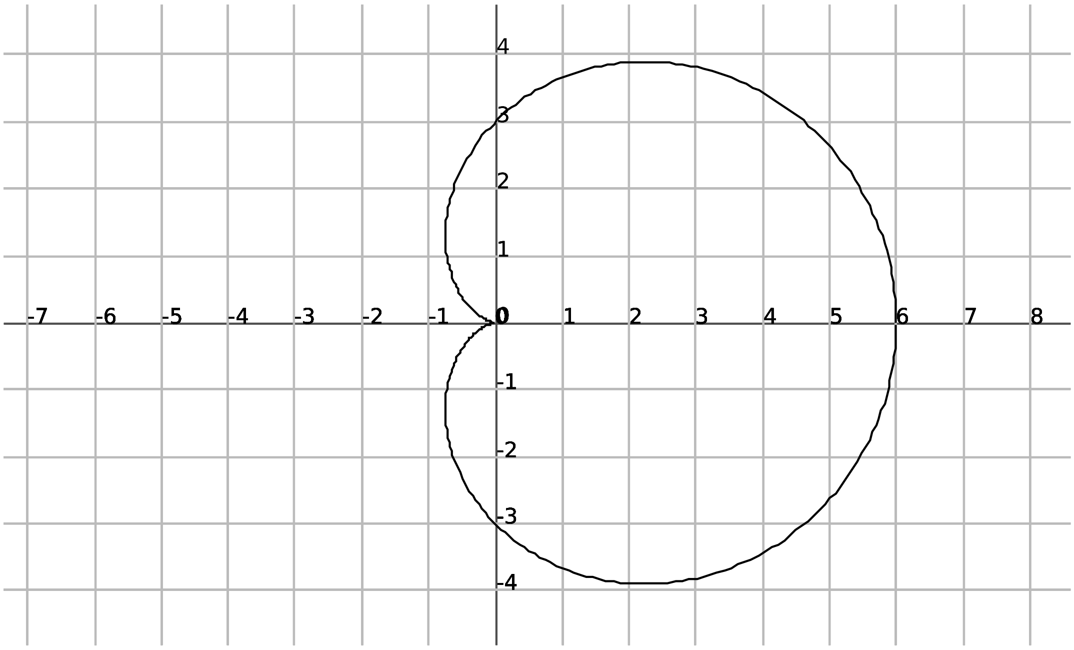


- $\boxed{r = 2 + 2\cos\theta}$ cardioid (think about $r = 2 - 2\cos\theta$)



Note: These cardioids cycle through and close one full loop as θ ranges from say $\theta = 0$ to $\theta = 2\pi$.

- $\boxed{r = 3 + 3\cos\theta}$ cardioid (think about $r = 3 - 3\cos\theta$)



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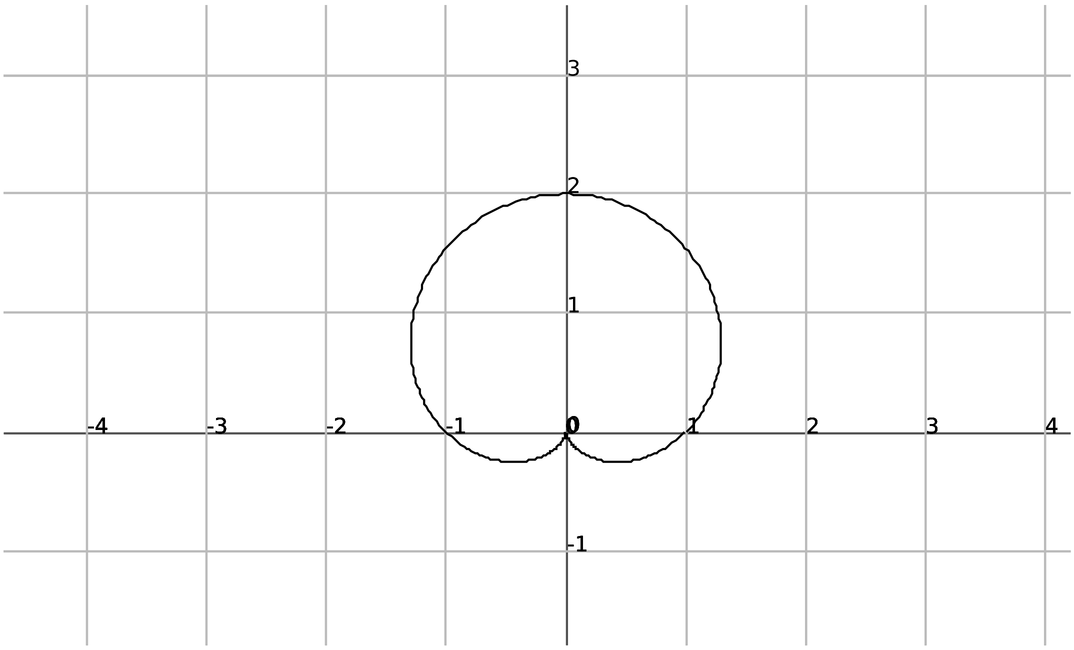
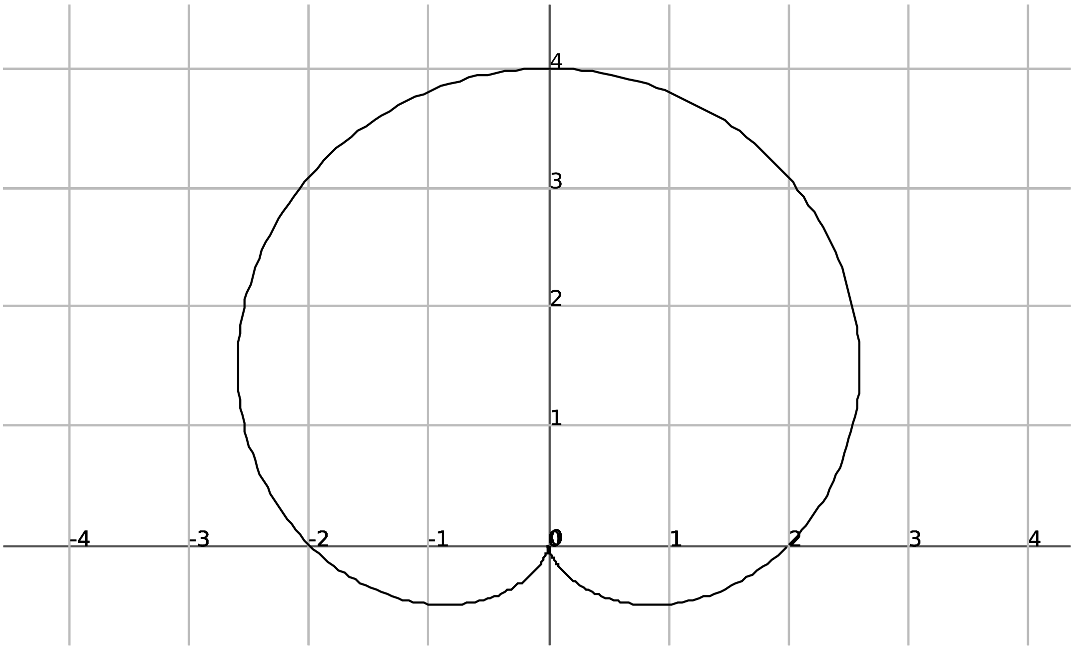
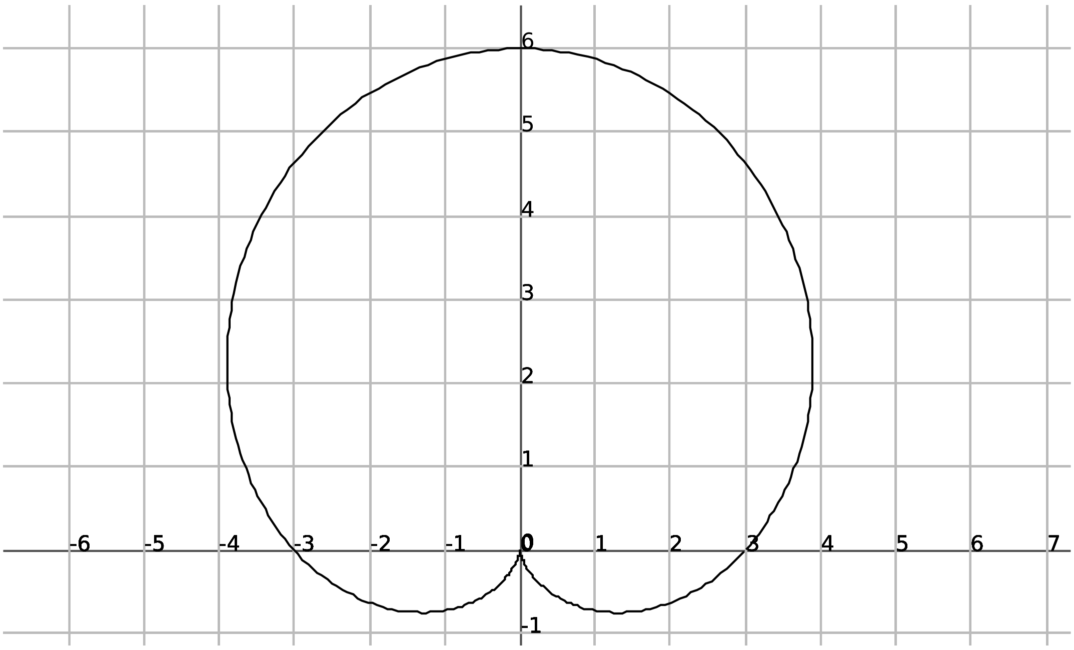
- $\boxed{r = 1 + \sin\theta}$ cardioid (think about $r = 1 - \sin\theta$)



- $\boxed{r = 2 + 2\sin\theta}$ cardioid (think about $r = 2 - 2\sin\theta$)



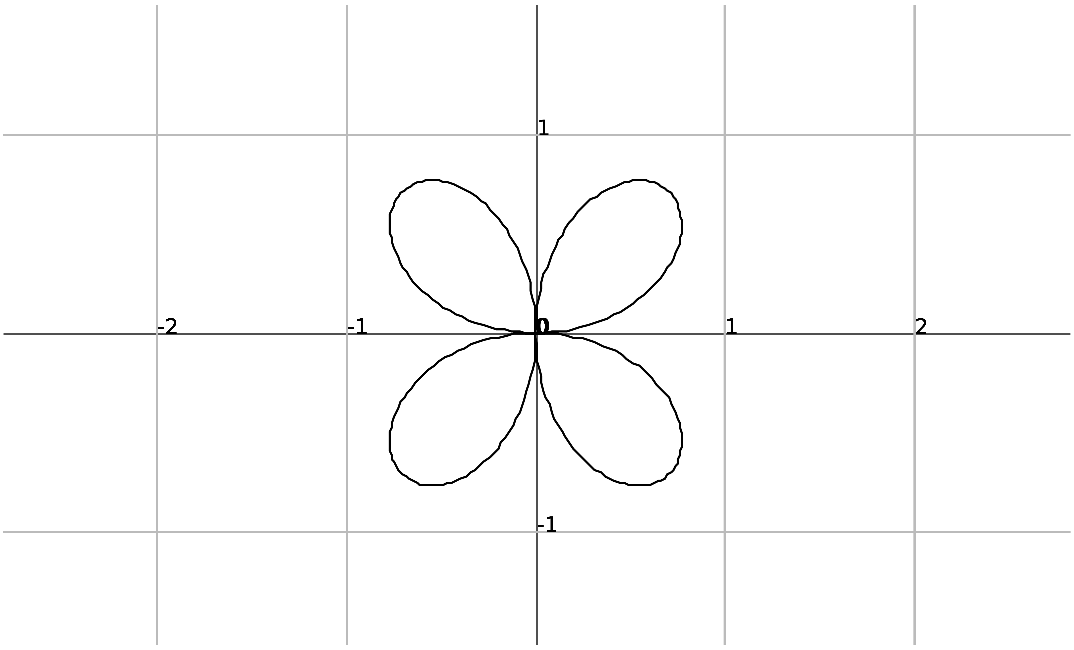
- $\boxed{r = 3 + 3\sin\theta}$ cardioid (think about $r = 3 - 3\sin\theta$)



You should also know how to sketch say $r = 3$ or $r = 1$. Recall the area formula for polar curves or intersections of 2 curves. The area is given by

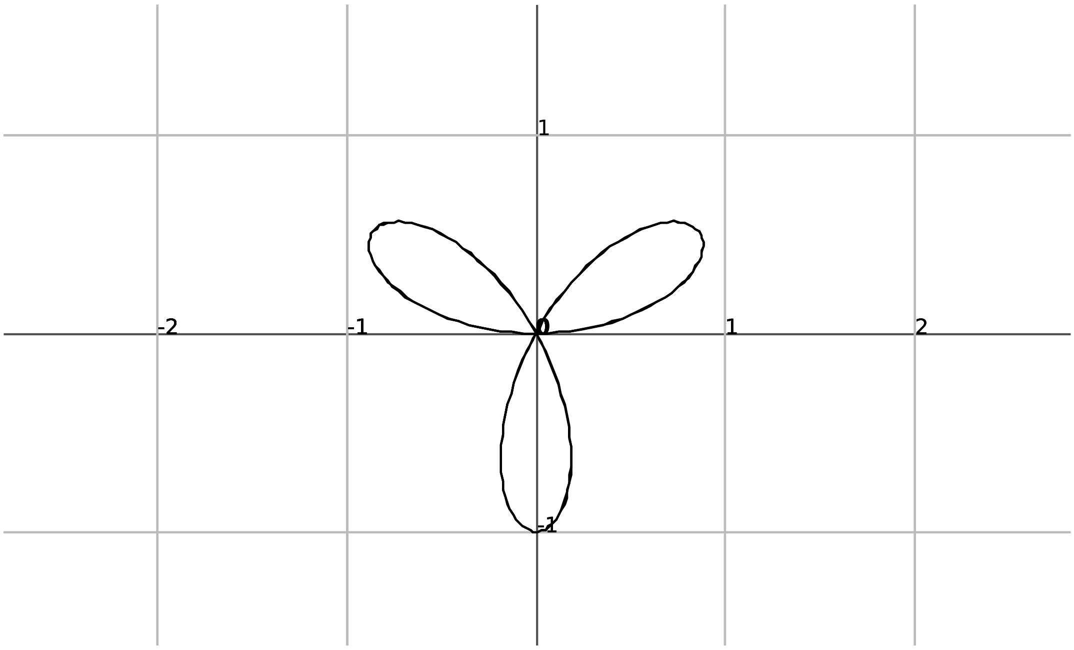
$$A = \int_{\theta=\alpha}^{\theta=\beta} \frac{1}{2} r^2 \, d\theta \quad \text{or} \quad A = \int_{\theta=\alpha}^{\theta=\beta} \frac{1}{2} \left((\text{outer polar curve})^2 - (\text{inner polar curve})^2 \right) \, d\theta \ .$$

- $\boxed{r = \sin 2\theta}$ 4-leaved rose



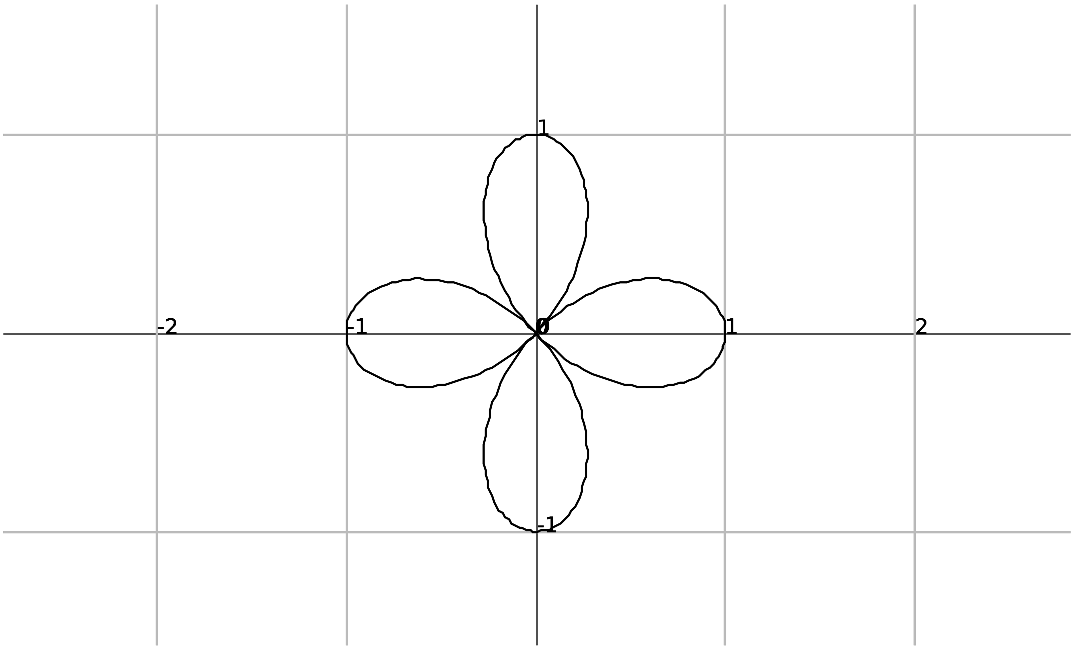
Think about how you would compute the area of the region enclosed by one loop of this polar curve. What angles determine one closed loop? Think about how the graph of $y = \sin(2x)$ might help you sketch this polar curve.

- $\boxed{r = \sin 3\theta}$ 3-leaved rose



Question: Why are there only 3 petals for this graph and 4 for the one above?

- $\boxed{r = \cos 2\theta}$ 4-leaved rose



Question: Why are these sketches for the cosine version rotated?

- $\boxed{r = \cos 3\theta}$ 3-leaved rose