

Homework #8

Due **FRIDAY, March 3rd** in Gradescope by 11:59 pm ET

Goal: Exploring Integrals using Secant Trig Substitutions, some Complete the Square algebra and also Partial Fractions Decomposition (Proper cases).

FIRST: Read through and understand the following two Examples.

Recall: IF the denominator **FACTORS**, go use PFD; otherwise try Complete the Square algebra to start

Ex:

$$\begin{aligned} \int \frac{6}{x^2 + 2x + 8} dx &\stackrel{\text{complete square}}{=} \int \frac{6}{(x+1)^2 + 7} dx = 6 \int \frac{1}{u^2 + 7} du \\ &= 6 \left(\frac{1}{\sqrt{7}} \right) \arctan \left(\frac{u}{\sqrt{7}} \right) + C = \boxed{\frac{6}{\sqrt{7}} \arctan \left(\frac{x+1}{\sqrt{7}} \right) + C} \end{aligned}$$

$$\begin{array}{l} u = x + 1 \\ du = dx \end{array}$$

Ex:

$$\begin{aligned} \int_{-3}^1 \frac{6}{x^2 + 2x - 8} dx &\stackrel{\text{factor}}{=} \int_{-3}^1 \frac{6}{(x-2)(x+4)} dx \stackrel{\text{PFD}}{=} \int_{-3}^1 \frac{1}{x-2} - \frac{1}{x+4} dx \\ &= \ln|x-2| - \ln|x+4| \Big|_{-3}^1 = \ln|-1| - \ln 5 - (\ln|-5| - \ln 1) \\ &= 0 - \ln 5 - \ln 5 + 0 = \boxed{-2 \ln 5} \end{aligned}$$

Partial Fractions Decomposition:

$$\frac{6}{(x-2)(x+4)} = \frac{A}{x-2} + \frac{B}{x+4}$$

Clearing the denominator yields:

$$6 = A(x+4) + B(x-2)$$

$$6 = Ax + 4A + Bx - 2B$$

$$6 = (A+B)x + (4A-2B)$$

$$\text{so that } A+B=0 \text{ and } 4A-2B=6$$

$$\text{Solve for } A=1 \text{ and } B=-1$$

Continue to NEXT Page for HW problems.

Compute each of the following Integrals. Simplify when possible.

$$1. \int \frac{\sqrt{x^2 - 9}}{x^3} dx$$

$$2. \int \frac{1}{x^2 \sqrt{x^2 - 16}} dx$$

$$\uparrow \text{ sub } \begin{array}{l} x = 3 \sec \theta \\ dx = 3 \sec \theta \tan \theta d\theta \end{array}$$

$$\uparrow \text{ sub } \begin{array}{l} x = 4 \sec \theta \\ dx = 4 \sec \theta \tan \theta d\theta \end{array}$$

$$\uparrow \text{ hint: } \frac{\tan^2 \theta}{\sec^2 \theta} = \sin^2 \theta \text{ why?}$$

$$3. \int \frac{1}{\sqrt{4 - 4x - x^2}} dx$$

$$4. \int_{-1}^1 \frac{1}{x^2 + 4x + 7} dx$$

$$5. \int \sqrt{3 - 2x - x^2} dx$$

$$6. \int \frac{x + 4}{x^2 + 2x + 5} dx$$

$$7. \int_3^5 \frac{6}{x^2 - 4x + 7} dx$$

$$8. \int_0^3 \frac{6}{x^2 - 4x - 5} dx$$

$$9. \int_0^1 \frac{x - 4}{x^2 - 5x + 6} dx$$

$$10. \int \frac{\arctan x}{x^2} dx = \int \arctan x \cdot (x^{-2}) dx$$

$$11. \int_{\ln 2}^{\ln 5} \frac{2e^x}{e^{2x} - 1} dx$$

$$12. \int \frac{10}{(x - 1)(x^2 + 9)} dx$$

$$\uparrow \text{ hint: } \begin{array}{l} \frac{2}{u^2 - 1} = \frac{2}{(u - 1)(u + 1)} \\ = \frac{A}{u - 1} + \frac{B}{u + 1} \end{array}$$

$$\uparrow \text{ use PFD } \frac{10}{(x - 1)(x^2 + 9)} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 9}$$

REGULAR OFFICE HOURS

Monday: None this week

6:00–7:30 pm TA Admire, SMUDD 204

Tuesday: None this week

6–7:30 pm TA Admire, SMUDD 204

Wednesday: None this week

7:30–9:00 pm TA Aidee, SMUDD 204

Thursday: TBA

6:00–7:30 pm TA Ali, SMUDD 204

7:30–9:00 pm TA Aidee, SMUDD 204

Friday: 12:00–2:00 pm

6:00–7:30 pm TA Ali, SMUDD 204

Please e-mail with questions/concerns: dbenedetto@amherst.edu

Either Start early or don't start late.

Thanks for working hard. I appreciate it!