

Homework #4

Due Friday, February 10th in Gradescope by 11:59 pm ET

Goal: Reviewing Inverse Trigonometric Functions and Limits (no L'Hopital's Rule yet)**FIRST:** Read through and understand the following Examples.

Ex:

$$\begin{aligned} \int_e^{e^3} \frac{1}{x(3+(\ln x)^2)} dx &= \int_1^3 \frac{1}{3+u^2} du \stackrel{\text{a-rule}}{=} \frac{1}{\sqrt{3}} \arctan\left(\frac{u}{\sqrt{3}}\right) \Big|_1^3 \\ &= \frac{1}{\sqrt{3}} \left(\arctan\left(\frac{3}{\sqrt{3}}\right) - \arctan\left(\frac{1}{\sqrt{3}}\right) \right) \\ &= \frac{1}{\sqrt{3}} \left(\frac{\pi}{3} - \frac{\pi}{6} \right) = \frac{1}{\sqrt{3}} \left(\frac{2\pi}{6} - \frac{\pi}{6} \right) = \boxed{\frac{\pi}{6\sqrt{3}}} \end{aligned}$$

$$\begin{aligned} u &= \ln x \\ du &= \frac{1}{x} dx \end{aligned}$$

$$\begin{aligned} x = e &\Rightarrow u = \ln e = 1 \\ x = e^3 &\Rightarrow u = \ln e^3 = 3 \end{aligned}$$

Ex:

$$\int \frac{e^{3x}}{4+e^{3x}} dx = \frac{1}{3} \int \frac{1}{u} du = \frac{1}{3} \ln |u| + C = \boxed{\frac{1}{3} \ln |4+e^{3x}| + C}$$

$$\begin{aligned} u &= 4+e^{3x} \\ du &= 3e^{3x} dx \\ \frac{1}{3} du &= e^{3x} dx \end{aligned}$$

Ex:

$$\int \frac{e^{3x}}{4+e^{6x}} dx = \int \frac{e^{3x}}{4+(e^{3x})^2} dx \text{ look for the perfect square to ready for arctan rule}$$

$$\frac{1}{3} \int \frac{1}{4+u^2} du \stackrel{\text{a-rule}}{=} \frac{1}{3} \left(\frac{1}{2} \arctan\left(\frac{u}{2}\right) \right) + C = \boxed{\frac{1}{6} \arctan\left(\frac{e^{3x}}{2}\right) + C}$$

$$\begin{aligned} u &= e^{3x} \\ du &= 3e^{3x} dx \\ \frac{1}{3} du &= e^{3x} dx \end{aligned}$$

Next, Compute each of the following Integrals. Simplify.

$$\begin{array}{lll} 1. \int_2^{2\sqrt{3}} \frac{1}{\sqrt{16-x^2}} dx & 2. \int_0^{\ln 3} \frac{e^x}{3+e^{2x}} dx & 3. \int_0^{\ln \sqrt{3}} \frac{e^x}{\sqrt{4-e^{2x}}} dx \\ 4. \int_4^{4\sqrt{3}} \frac{1}{16+x^2} dx & 5. \int \frac{x}{\sqrt{1-x^4}} dx & 6. \int \frac{x^2}{x^2+4} dx \\ 7. \int \frac{2x^2+5}{x^2+1} dx & 8. \int \frac{1}{(1+x^2)(5+(\arctan x)^2)} dx & \\ 9. \int_3^9 \frac{1}{\sqrt{x}(x+9)} dx & 10. \int \frac{x^2+x+1}{x^2+4} dx & \end{array}$$

Compute each of the following Limits. Simplify. Use arrows to justify the size arguments.

$$\begin{array}{ll} 11. \lim_{x \rightarrow 5^+} \frac{1}{x-5} & 12. \lim_{x \rightarrow 5^-} \frac{1}{x-5} \\ 13. \lim_{x \rightarrow 8^+} \ln|x-8| & 14. \lim_{x \rightarrow 8^-} \ln|x-8| \\ 15. \lim_{x \rightarrow 3^+} e^{\frac{2}{x-3}} & 16. \lim_{x \rightarrow 3^-} e^{\frac{2}{x-3}} \\ 17. \lim_{x \rightarrow \infty} \ln \left(1 - \arctan \left(\frac{5}{x^4} \right) \right) & 18. \lim_{x \rightarrow \infty} \ln \left(\frac{\pi}{2} - \arctan x \right) \\ 19. \lim_{x \rightarrow 4^-} \ln |\ln|x-4|| & 20. \lim_{x \rightarrow 0^+} \arctan \left(\frac{\ln x}{5} \right) \end{array}$$

21. Present two different methods to Prove that $\int \frac{1}{4+x^2} dx = \frac{1}{2} \arctan \left(\frac{x}{2} \right) + C$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6:00–7:30 pm TA Admire, SMUDD 204

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Admire, SMUDD 204

Wednesday: 1:00–3:00 pm

7:30–9:00 pm TA Aidee, SMUDD 204

Thursday: none for Professor

6:00–7:30 pm TA Ali, SMUDD 204

7:30–9:00 pm TA Aidee, SMUDD 204

Friday: 12:00–2:00 pm

6:00–7:30 pm TA Ali, SMUDD 204

- Please stop by for help! Try to attend at least one office hour for me and at least one for the Math Fellows each week.
- You can also find help at the Math Fellow (Aidee, Ali, or Admire) sessions.