Math 121, Section(s) 01, Spring 2023

Homework #10

Due **Friday, October 14th** in Gradescope by 11:59 pm ET

Goal: Exploring Limits of Infinite Sequences. We may also need L'Hôpital's Rule to finish some of the limits at hand.

FIRST: Read through and understand the following 5 Examples.

Determine whether the given sequence Converges or Diverges. If it converges, find the Limit.

Ex: $\left\{\frac{\ln n}{n^3}\right\}_{n=1}^{\infty} \hookrightarrow \lim_{n\to\infty}\frac{\ln n}{n^3}\overset{\frac{\infty}{\infty}}{} = \lim_{x\to\infty}\frac{\ln x}{x^3} \overset{\text{L'H}}{=} \lim_{x\to\infty}\frac{\frac{1}{x}}{3x^2} = \lim_{x\to\infty}\frac{1}{3x^3} = \boxed{0}$ Converges

Ex: $\left\{\frac{e^n}{n^2}\right\}_{n=1}^{\infty} \hookrightarrow \lim_{n\to\infty}\frac{e^n}{n^2}\overset{\frac{\infty}{\infty}}{} = \lim_{x\to\infty}\frac{e^x}{x^2} \overset{\text{L'H}}{=} \lim_{x\to\infty}\frac{e^x}{2x} = \lim_{x\to\infty}\frac{e^x}{2} = \boxed{\infty}$ Diverges

Ex: $\left\{\frac{4-9n^3}{5n^3+8n^2-7n-6}\right\}_{n=1}^{\infty}$ Note: Can switch to the Related Function in x and use L'H Rule or ... Quicker ...

$$\hookrightarrow \lim_{n\to\infty}\frac{4-9n^3}{5n^3+8n^2-7n-6}\,\frac{\left(\frac{1}{n^3}\right)}{\left(\frac{1}{n^3}\right)} = \lim_{n\to\infty}\frac{\frac{4}{n^3}-9}{5+\frac{8}{n}-\frac{7}{n^2}-\frac{6}{n^3}} = \boxed{-\frac{9}{5}}\text{ Converges}$$

Ex: $\left\{\left(1-\sin\left(\frac{6}{n^3}\right)\right)^{n^3}\right\}_{n=1}^{\infty} \hookrightarrow \lim_{n\to\infty}\left(1-\sin\left(\frac{6}{n^3}\right)\right)^{n^3} \overset{1^\infty}{=} \lim_{x\to\infty}\left(1-\sin\left(\frac{6}{x^3}\right)\right)^{x^3}$

$$= e^{\lim_{x\to\infty}\ln\left[\left(1-\sin\left(\frac{6}{x^3}\right)\right)^{x^3}\right]} = e^{\lim_{x\to\infty}x^3\ln\left(1-\sin\left(\frac{6}{x^3}\right)\right)} \overset{\infty\cdot 0}{=} e^{\lim_{x\to\infty}\frac{\ln\left(1-\sin\left(\frac{6}{x^3}\right)\right)}{\frac{1}{x^3}}\,\frac{0}{0}}$$

$$\overset{\text{L'H}}{=} e^{\lim_{x\to\infty}\frac{\frac{1}{1-\sin\left(\frac{6}{x^3}\right)}\cdot\left(-\cos\left(\frac{6}{x^3}\right)\right)\cdot\left(-\frac{18}{x^4}\right)}{-\frac{3}{x^4}}} = e^{-6} = \boxed{\frac{1}{e^6}}\text{ Converges}$$

Continue to NEXT Page

Ex:

$$\left\{\frac{(3n-1)!}{(3n+1)!}\right\}_{n=1}^{\infty}$$

$$\hookrightarrow \lim_{n\to\infty}\frac{(3n-1)!}{(3n+1)!} = \lim_{n\to\infty}\frac{\cancel{(3n-1)!}}{(3n+1)(3n)\cancel{(3n-1)!}} = \lim_{n\to\infty}\frac{1}{\underbrace{(3n+1)(3n)}_{\to\infty}} \to 0 = \boxed{0} \text{ Converges}$$

Next do the following HW problems.

List the first five terms of the Sequence. (Start with $n = 1$)

1. $a_n = \dfrac{(-1)^{n-1}}{5^n}$
2. $a_n = \dfrac{1}{(n+1)!}$
3. $a_n = \dfrac{(-1)^n\, n^2}{n+1}$

Determine whether the given sequence Converges or Diverges. If it converges, find the Limit. Justify, no guessing here.

4. $\left\{\dfrac{n}{n+1}\right\}_{n=1}^{\infty}$
5. $\left\{\dfrac{5n^2+3}{2n^2-7n}\right\}_{n=1}^{\infty}$
6. $\left\{\dfrac{3n^4-n-5}{7n^4+n^2-9}\right\}_{n=1}^{\infty}$
7. $\left\{\dfrac{\tan^{-1} n}{n}\right\}$
8. $\left\{\dfrac{n^2}{e^n}\right\}$
9. $\left\{n\sin\left(\dfrac{1}{n}\right)\right\}$
10. $\left\{\dfrac{(\ln n)^2}{n}\right\}_{n=1}^{\infty}$
11. $\left\{\dfrac{n^{99}}{\ln n}\right\}_{n=2}^{\infty}$
12. $\left\{\dfrac{\ln(99)}{n^{99}}\right\}$
13. $\left\{\left(1+\dfrac{1}{n}\right)^n\right\}_{n=1}^{\infty}$
14. $\left\{\left(1-\dfrac{5}{n^6}\right)^{n^6}\right\}_{n=1}^{\infty}$
15. $\left\{\left(1-\arcsin\left(\dfrac{3}{n^2}\right)\right)^{n^2}\right\}$
16. $\{\ln(2n^2+1)-\ln(n^2+1)\}$
17. $\left\{\dfrac{(n+3)!}{(n+1)!}\right\}_{n=1}^{\infty}$
18. $\left\{\dfrac{(2n-1)!}{(2n+1)!}\right\}$
19. $\left\{\cos^2\left(\dfrac{\pi n^6+6}{6n^6+1}\right)\right\}_{n=1}^{\infty}$
20. $\left\{\arctan\left(\dfrac{5n^7+1}{5n^7+7}\right)\right\}_{n=1}^{\infty}$

REGULAR OFFICE HOURS

Monday: None this week

6:00–7:30 pm TA Admire, SMUDD 204

Tuesday: None this week

6–7:30 pm TA Admire, SMUDD 204

Wednesday: None this week

7:30–9:00 pm TA Aidee, SMUDD 204

Thursday: TBA

6:00–7:30 pm TA Ali, SMUDD 204

7:30–9:00 pm TA Aidee, SMUDD 204

Friday: 12:00–2:00 pm

6:00–7:30 pm TA Ali, SMUDD 204

dig deep, check notation, reference, justify, search, clarify...

challenge to everyone this week, get help on a challenging problem in office hours with me or a Math Fellow