Math 121 Exam #2 Mar 31-April 2

Due Sunday, April 2, 2023 in Gradescope by 11:59 pm

- This is **NOT** an Open Notes Exam. You can **NOT** access any materials, homeworks problems, lecture notes, etc. You may use one 5x7 Cheat Sheet.
- There is **NO** *Open Internet* access allowed. Do **NOT** use any online sources.
- You are not allowed to discuss these problems with anyone, including Math Fellows.
- Submit your final work in Gradescope in the Exam 2 entry.
- Please *show* all of your work and *justify* all of your answers. No Calculators.

1. [40 Points] Compute the following **Improper** integrals. Simplify all answers. Justify.

(a) $\displaystyle\int_0^e x^2 \ln\left(x^2\right)\, dx = \boxed{\frac{4e^3}{9}}$

(b) $\displaystyle\int_e^\infty \frac{\ln x}{x^2}\, dx = \int_e^\infty (\ln x)\, x^{-2}\, dx = \boxed{\frac{2}{e}}$

(c) $\displaystyle\int_{-\infty}^{-3} \frac{8-x}{x^2+2x+5}\, dx = \boxed{\infty}$

(d) $\displaystyle\int_{-4}^{-3} \frac{8-x}{x^2+2x-8}\, dx = \boxed{-\infty}$

2. [10 Points] Use the Integral Test to determine if $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4n+7}$ **Converges** or **Diverges**.

Note: You do **not** have to check the 3 pre-conditions.

3. [35 Points] Determine whether each of the given series **Converges** or **Diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} n^5+n^4+n^3+n^2+n+1$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+5)^8}{\ln(n+5)}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^8}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln 5}{(n+5)^8} + \frac{(-1)^n \cdot 8}{5^{2n+1}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \arctan\left(\frac{n^8+\sqrt{3}}{\sqrt{3}\, n^8+5}\right)$

4. [10 Points] Use the Absolute Convergence Test to show that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5+n^4+n^3+n^2+n+1}$ **Converges**.

5. [30 Points] Determine whether the given series is **Absolutely Convergent**, **Conditionally Convergent**, or **Divergent** or follow the description. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^5+5n+8}{n^8+5}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ n^5 \cdot n^n \cdot n!}{(2n+1)!}$

(c) Give an example of an **Alternating Series** which is **Conditionally Convergent**.

You **cannot** choose just $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ or $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \text{ constant}}{n^p}$.

Continue on to Prove that this series is Conditionally Convergent.

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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BONUS: Prove that the Sequence $\displaystyle\left\{\frac{(\ln n)\cdot 2^n \cdot (n!)^2}{n^{2n}\cdot (3n)!}\right\}_{n=1}^{\infty}$ Converges.