Math 121 Midterm Exam #1 September 30, 2022

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\arctan(\sqrt{3})$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. Limits [30 Points total, 10 Points each] Evaluate each of the following. Please justify/simplify.

(a) Show that $\lim_{x\to 0} \dfrac{\cos(3x) - \arctan(2x) + 2x - 1}{e^{-4x} - 1 + 4x} = \boxed{-\dfrac{9}{16}}$

(b) Show that $\lim_{x\to 0^+} x \ln x = \boxed{0}$

(c) Show that $\lim_{x\to\infty} \left(1 - \arcsin\left(\dfrac{2}{x^6}\right)\right)^{x^6} = \boxed{e^{-2}}$

Integrals [34 Points total] Compute each of the following Definite Integrals. Please justify/simplify.

2. Show that $\displaystyle\int_{-2}^{2} \sqrt{4 - x^2}\ dx = \boxed{2\pi}$

3. Show that $\displaystyle\int_{0}^{\ln\sqrt{3}} \frac{e^x}{\sqrt{4 - e^{2x}}}\ dx = \boxed{\frac{\pi}{6}}$

4. Show that $\displaystyle\int_{e}^{e^3} \frac{1}{x\left[3 + (\ln x)^2\right]}\ dx = \boxed{\frac{\pi}{6\sqrt{3}}}$

More Integrals [36 Points total] Compute the following Indefinite Integrals. Please justify/simplify.

5. Compute $\displaystyle\int x \arcsin x\ dx$

6. Compute $\displaystyle\int \frac{1}{(9 + x^2)^{\frac{7}{2}}}\ dx$ Hint: $3^6 = 729$

7. Compute $\displaystyle\int x^7 \cdot \ln\left(x^3\right)\ dx$