

**What you need to know for Exam 3-Spring 2022**

You should know Sections 11.8–11.10, Power Series Applications, and Sections 10.1–10.2, This exam will not explicitly cover the material from earlier sections, but of course it will still be assumed that you know how to deal with convergence of infinite series, exponentials, logarithms, trig and inverse trig functions, L'Hôpital's rule, substitution, and so on. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- 11.8: Power Series. Know the definition, and remember that it includes not just series of the form  $\sum c_n x^n$  but also series like  $\sum c_n (x - a)^n$ , where  $a$  is a constant, called the **center** of the power series. Know how to find the radius and interval of convergence of a given power series. Remind yourself what the 3 options are for the (domain) intervals of convergence. See Theorem 3. You will need to recall the works of Sections 11.2-11.6 in order to analyze specific series that arise at each endpoint.
- 11.9: Power Series Representations of Functions. Know how to use power series you already know (like that for  $1/(1 - x)$ , or (in later sections)  $e^x$ ,  $\cos x$ ,  $\sin x$ ) to find power series for other functions by the following five operations: (1) substituting monomials like  $-3x$  or  $2x^3$  for  $x$ , or (2) multiplying by a polynomial, or (3) adding two power series (centered at the same center), or (4) integrating (usually followed by evaluating at  $x = a$  to find the constant  $C$  from integration), or (5) differentiating. Know how each of these operations may affect the Radius of Convergence. See the Theorem.
- 11.10: Taylor and MacLaurin Series. Know the following:
  - Computing Taylor/MacLaurin Series using the Definition (Chart Method)
  - Our 6 memorized MacLaurin Series
  - Applications, including new (indefinite) integrals, new sums (beyond geometric), ~~new higher-order derivatives~~, and estimates for values or (definite) integrals using Alternating Series Estimation Theorem, and finally Limits using Series.
- 10.1: Parametric Equations. Know what a parametric curve is, and be able to sketch such a curve (including the direction it is traversed) by plotting points, or by eliminating the parameter. (The latter is either solving for  $y$  in terms of  $x$ , or solving for  $x$  in terms of  $y$ , or if sines and cosines are involved, you can try squaring the terms and see if the parametric equations satisfy some equation of a circle, say.)
- 10.2: Calculus with Parametric Curves. Know and be able to use the formula for slopes of tangent lines for parametric curves. Know how to find the arclength, ~~and surface area for rotation about the  $x$  or  $y$  axis~~. ~~(Both formulas, even the one not stated in book, but rather I gave that in class)~~. Look for algebra assistance (perfect squares?) in the complicated square root chunk.
- 5.2–5.3: ~~Volumes of Revolution~~ ~~NOT COVERED Spring 2020~~

### Some things you don't need to know

- Section 11.10: Taylor Series Remainder material
- Sections 11.10: Binomial Series
- All of Section 11.11 except for  $n^{\text{th}}$  Taylor Polynomials
- Chapter 10: Anything about graphing calculators
- Section 10.1: All the stuff about **families** of parametric curves.
- Section 10.2: Area for Parametric Equations
- Sections 10.3-10.4: Polar Coordinates (for Exam 3 anyhow)
- Chapter 10: The specific names for various curves (Astroids...)

### Tips

- Know the three possibilities for Intervals of Convergence. If it the finite case, make sure to manually check convergence at the endpoints. Understand why the Ratio Test is inconclusive at the endpoints.

Remember  $\lim_{n \rightarrow \infty} \left(\frac{n+1}{n}\right)^n = e$  and  $\lim_{n \rightarrow \infty} \left(\frac{n}{n+1}\right)^n = \frac{1}{e}$

- For power series representations: if you're asked to find the Taylor series (or power series rep) of a function  $f(x)$ , or even a Taylor polynomial, your first attempt should be to write  $f$  in terms of the basic functions  $1/(1-x)$ ,  $e^x$ ,  $\cos x$ ,  $\sin x$  using only the operations that are good for power series (see the 11.9 discussion above). If that doesn't seem to work, then you can try to blast out the Taylor series by taking all those derivatives, using the *chart method*. It also takes much more time. Finding the MacLaurin series for  $x^4 \cos x$  takes about five seconds if you just multiply the series for  $\cos x$  by  $x^4$ ; but it's a horrendous and time-consuming mess to compute by blasting out all the derivatives.)

- If you're asked for the **sum** of a series, on Exam 2 that meant it'd have to be something like a geometric series ~~or a telescoping series~~. Now we have a third possibility: that you can get that series by plugging in  $x$  equals some specific number in some specific power series. For example,

$\sum_{n=0}^{\infty} \frac{5^{n-2}}{2^{2n} \cdot n!}$  can be rewritten as  $\frac{1}{25} \sum_{n=0}^{\infty} \frac{(5/4)^n}{n!} = \frac{1}{25} e^{5/4}$ . Similarly,  $\sum_{n=0}^{\infty} \frac{(-1)^n 7^n}{n!} = e^{-7}$ .