Extra Examples of Limits

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Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

1. $$\lim_{x\to 0} \frac{\arcsin x + \cos(3x) - e^x}{\arctan(3x) + x^2 - \sinh(3x)} \overset{\left(\frac{0}{0}\right)\text{L'H}}{=} \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}} - 3\sin(3x) - e^x}{\frac{3}{1+9x^2} + 2x - 3\cosh(3x)} \left(\frac{0}{0}\right)$$

$$\overset{\text{L'H}}{=} \lim_{x\to 0} \frac{-\frac{1}{2(1-x^2)^{\frac{3}{2}}}(-2x) - 9\cos(3x) - e^x}{-\frac{3}{(1+9x^2)^2}(18x) + 2 - 9\sinh(3x)}$$

$$= \lim_{x\to 0} \frac{\frac{x}{(1-x^2)^{\frac{3}{2}}} - 9\cos(3x) - e^x}{-\frac{54x}{(1+9x^2)^2} + 2 - 9\sinh(3x)} = \frac{-9-1}{2} = \frac{-10}{2} = \boxed{-5}$$

2. $$\lim_{x\to 0} \frac{\ln(1-x) + x}{\cosh(4x) - \arctan(3x) - e^{-3x}} \left(\frac{0}{0}\right) \overset{\text{L'H}}{=} \lim_{x\to 0} \frac{-\frac{1}{1-x} + 1}{4\sinh(4x) - \frac{3}{1+9x^2} + 3e^{-3x}} \left(\frac{0}{0}\right)$$

$$\overset{\text{L'H}}{=} \lim_{x\to 0} \frac{-\frac{1}{(1-x)^2}}{16\cosh(4x) + \frac{3(18x)}{(1+9x^2)^2} - 9e^{-3x}} = -\frac{1}{16-9} = \boxed{-\frac{1}{7}}$$

3. $$\lim_{x\to 0^+} (1-3\sin x)^{\frac{1}{x}} \overset{1^\infty}{=} e^{\lim_{x\to 0^+} \ln\left[(1-3\sin x)^{\frac{1}{x}}\right]}$$

$$= e^{\lim_{x\to 0^+} \frac{\ln(1-3\sin x)}{x}} \left(\frac{0}{0}\right) \overset{\text{L'H}}{=} e^{\lim_{x\to 0^+} \frac{\frac{1}{1-3\sin x}(-3\cos x)}{1}} = \boxed{e^{-3}}$$

4. $$\lim_{x\to\infty} (\ln x)^{\frac{3}{x}} \overset{\infty^0}{=} \lim_{x\to\infty} e^{\ln\left((\ln x)^{\frac{3}{x}}\right)} = e^{\lim_{x\to\infty} \ln\left((\ln x)^{\frac{3}{x}}\right)} = e^{\lim_{x\to\infty} \left(\frac{3}{x}\right)\ln(\ln x)}$$

$$e^{\lim_{x\to\infty} \left(\frac{3\ln(\ln x)}{x}\right)} \overset{\frac{\infty}{\infty}}{\overset{\text{L'H}}{=}} e^{\lim_{x\to\infty} \left(\frac{\left(\frac{3}{\ln x}\right)\left(\frac{1}{x}\right)}{1}\right)} = e^{\lim_{x\to\infty} \left(\frac{3}{x\ln x}\right)} = e^0 = \boxed{1}$$

5. $$\lim_{x\to\infty}\left(1-\arcsin\left(\frac{5}{x^4}\right)\right)^{3x^4} \overset{1^\infty}{=} e^{\lim\limits_{x\to\infty}\ln\left[\left(1-\arcsin\left(\frac{5}{x^4}\right)\right)^{3x^4}\right]}$$

$$= e^{\lim\limits_{x\to\infty} 3x^4\ln\left(1-\arcsin\left(\frac{5}{x^4}\right)\right)} \overset{\infty\cdot 0}{=} e^{\lim\limits_{x\to\infty}\dfrac{3\ln\left(1-\arcsin\left(\frac{5}{x^4}\right)\right)}{\frac{1}{x^4}}^{\left(\frac{0}{0}\right)}}$$

$$\overset{\text{L'H}}{=} e^{\lim\limits_{x\to\infty}\dfrac{\left(\frac{3}{1-\arcsin\left(\frac{5}{x^4}\right)}\right)\left(-\frac{1}{\sqrt{1-\left(\frac{5}{x^4}\right)^2}}\right)\left(-\frac{20}{x^5}\right)}{-\frac{4}{x^5}}}$$

$$= e^{\lim\limits_{x\to\infty}\left(\frac{3}{1-\arcsin\left(\frac{5}{x^4}\right)}\right)\left(-\frac{1}{\sqrt{1-\left(\frac{5}{x^4}\right)^2}}\right)(5)} = \boxed{e^{-15}}$$

6. $$\lim_{x\to\infty}\left(1-\arcsin\left(\frac{6}{x}\right)\right)^{x} \overset{1^\infty}{=} \lim_{x\to\infty} e^{\ln\left(\left(1-\arcsin\left(\frac{6}{x}\right)\right)^{x}\right)}$$

$$= e^{\lim\limits_{x\to\infty}\ln\left(\left(1-\arcsin\left(\frac{6}{x}\right)\right)^{x}\right)} = e^{\lim\limits_{x\to\infty} x\ln\left(1-\arcsin\left(\frac{6}{x}\right)\right)^{\infty\cdot 0}}$$

$$= e^{\lim\limits_{x\to\infty}\dfrac{\ln\left(1-\arcsin\left(\frac{6}{x}\right)\right)^{\left(\frac{0}{0}\right)}}{\frac{1}{x}}} \overset{\text{L'H}}{=} e^{\lim\limits_{x\to\infty}\dfrac{\left(\frac{1}{1-\arcsin\left(\frac{6}{x}\right)}\right)\left(-\frac{1}{\sqrt{1-\left(\frac{6}{x}\right)^2}}\right)\left(-\frac{6}{x^2}\right)}{-\frac{1}{x^2}}}$$

$$= e^{\lim\limits_{x\to\infty}\left(\frac{1}{1-\arcsin\left(\frac{6}{x}\right)}\right)\left(-\frac{1}{\sqrt{1-\left(\frac{6}{x}\right)^2}}\right)(6)} = \boxed{e^{-6}}$$