Homework #3

Due Wednesday, February 16th in Gradescope by 11:59 pm ET

Goal: Solidying Calculus for Inverse Sine and Inverse Tangent.

Differentiate the following functions. Simplify.

1.
$$f(x) = \tan^{-1}(x^2)$$

2.
$$f(x) = (\tan^{-1}(x))^2$$

3.
$$y = x \sin^{-1} x + \sqrt{1 - x^2}$$

1.
$$f(x) = \tan^{-1}(x^2)$$
 2. $f(x) = (\tan^{-1}(x))^2$
3. $y = x \sin^{-1} x + \sqrt{1 - x^2}$ 4. $f(x) = \ln\left(1 - \arcsin\left(\frac{2}{x^4}\right)\right)$

5. Find the value of the expression
$$\tan \left(\sin^{-1}\left(\frac{2}{3}\right)\right)$$

6. Simplify the expression
$$\sin(\tan^{-1} x)$$

7. Compute the Second Derivative for
$$f(x) = \arctan(2x)$$

8. Compute the Second Derivative for
$$f(x) = \arcsin(6x)$$

9. **Prove** that
$$\frac{d}{dx}\sin^{-1}(3x) = \frac{3}{\sqrt{1-9x^2}}$$

10. Prove that
$$\frac{d}{dx} \tan^{-1}(5x) = \frac{5}{1 + 25x^2}$$

11. Use Integration to **Justify** that
$$\int \frac{1}{3+x^2} dx = \frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right) + C$$

Compute each of the following Integrals. Simplify.

$$12. \int \frac{x^2}{x^2 + 1} \ dx$$

$$13. \int \frac{x+1}{x^2+1} \ dx$$

13.
$$\int \frac{x+1}{x^2+1} dx$$
 14. $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{8}{1+x^2} dx$

15.
$$\int_0^{\frac{1}{2}} \frac{\arcsin x}{\sqrt{1 - x^2}} \, dx$$

16.
$$\int \frac{1}{\sqrt{1-x^2} \cdot \sin^{-1} x} dx \quad 17. \int_1^3 \frac{1}{\sqrt{x}(1+x)} dx$$

17.
$$\int_{1}^{3} \frac{1}{\sqrt{x}(1+x)} \ dx$$

18.
$$\int_0^{\ln 3} \frac{e^x}{1 + e^x} \ dx$$

19.
$$\int_0^{\frac{1}{2}\ln 3} \frac{e^x}{1 + e^{2x}} \ dx$$

19.
$$\int_0^{\frac{1}{2}\ln 3} \frac{e^x}{1+e^{2x}} dx$$
 20. $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}} dx$

21.
$$\int_{3}^{3\sqrt{3}} \frac{1}{\sqrt{36 - x^2}} + \frac{1}{9 + x^2} dx$$

REGULAR OFFICE HOURS

Sunday: 6–7:30 pm TA Nico, SMUDD 207

Monday: 1:00–3:00 pm

6–7:30 pm TA Daksha, SMUDD 207

7:30–9:00 pm TA Karime, SMUDD 207

Tuesday: 12:00–4:00 pm

6-7:30 pm TA Ian, SMUDD 207

7:30–9:00 pm TA Nico, SMUDD 207

Wednesday: 1:00-3:00 pm

 $9-10:30~\mathrm{pm}$ TA Daksha, SMUDD 207

Thursday: none for Professor

6-7:30 pm TA Ian, SMUDD 207

7:30–9:00 pm TA Karime, SMUDD 207

Friday: 12:00–2:00 pm

- Please do not wait until the last night to start.
- Please stop by for help! Please try the homework before you come by though. Final Answer keys are posted on the webpage. Please do **not** look at them unless you have completed the problems. **They are not a replacement for my help or your understanding.**
- You can also find help at the Math Fellow (Nico, Ian, Karime or Daksha) sessions.