Math 121, Section 01, Spring 2022

Homework #16

Due **Wednesday, April 20th** in Gradescope by 11:59 pm ET

Goal: Exploring more of the Relationship between Power Series and functions, inlcuding differentiation and integration of Power Series.

For $\#1-4$, find the Power Series Representation for the following functions and determine the Radius of Convergence R.

1. $f(x) = \dfrac{1}{1+x^2}$

2. $f(x) = \dfrac{x^2}{x^4+16}$

3. $f(x) = \dfrac{1}{(1+x)^2}$ Hint: $\dfrac{1}{(1+x)^2} = \dfrac{d}{dx}\left(-\dfrac{1}{1+x}\right)$

4. $f(x) = \arctan(x^2)$

(a) First, Use this fact $\arctan(x^2) = \displaystyle\int \frac{2x}{1+x^4}\,dx$. Show that $C = 0$.

(b) Next, use *Substitution* into the known Series formula for $\arctan x$. Show (a)/(b) match.

5. Prove the Power Series Representation formula for $\ln(1+x)$. Yes, show that $C = 0$.

6. Prove the Power Series Representation formula for $\arctan x$. Yes, show that $C = 0$.

7. Find Power Series Representation for $\ln(5-x)$. Solve for C and the Radius R.

8. You do **not** need to state the Radius. Answers should be in Sigma notation $\displaystyle\sum_{n=0}^{\infty}$ here.

You may use the fact that $\boxed{\sin x = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}}$ without extra justification.

(a) Use the Definition to compute the MacLaurin Series for $F(x) = \cos x$.

(b) Use Differentiation to compute the Series for $F(x) = \cos x$.

(c) Use Integration to compute the Series for $F(x) = \cos x$.

Hints: yes, you should solve for $+C$. yes, C should equal 1. Show why $C = 1$.

REGULAR OFFICE HOURS

Sunday: 6–7:30 pm TA Nico, SMUDD 207

Monday: 1:00–3:00 pm

6–7:30 pm TA Daksha, SMUDD 207

7:30–9:00 pm TA Karime, SMUDD 207

Tuesday: 12:00–4:00 pm

6–7:30 pm TA Ian, SMUDD 207

7:30–9:00 pm TA Nico, SMUDD 207

Wednesday: 1:00-3:00 pm

9–10:30 pm TA Daksha, SMUDD 207

Thursday: none for Professor

6–7:30 pm TA Ian, SMUDD 207

7:30–9:00 pm TA Karime, SMUDD 207

Friday: 12:00–2:00 pm

Pay careful attention to details here.
Manipulating power series requires a balance
of memory and technical skill.