Math 121, Section 01, Spring 2022

Homework #13

Due **Wednesday, April 6th** in Gradescope by 11:59 pm ET

Goal: Exploring Convergence of Infinite Series. Focus on Alternating Series Test, and Ratio Test. We will also focus on fluency of training, using multiple tests.

1. Consider $\sum_{n=1}^{\infty} \frac{n+1}{n^2+4n+7}$. Use **two** Different methods, namely the Integral Test and the Limit Comparison Test, to prove that this series Diverges. *You can skip checking the Integral Test preconditions here this time. yay!*

Determine if the given Alternating series Converges or Diverges.

2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n+1}$

3. $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$

Determine if the given series is Absolutely Convergent or Divergent.

4. $\sum_{n=1}^{\infty} \frac{n}{5^n}$

5. $\sum_{n=1}^{\infty} \frac{(-3)^n}{(2n+1)!}$

6. $\sum_{n=1}^{\infty} \frac{n!}{100^n}$

7. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

8. $\sum_{n=1}^{\infty} \frac{n^{100}\ 100^n}{n!}$

9. $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

10. Consider the series $\sum_{n=1}^{\infty} \frac{\ln n}{n}$.

(a) Show that n^{th} Term Divergence Test is **Inconclusive**.

(b) Show that the Ratio Test is **Inconclusive**.

(c) Show that the series Diverges using the Integral Test. Skip checking the 3 preconditions here. **Note:** This is an example where the terms approach 0 but the series Diverges.

11. Prove that $\sum_{n=1}^{\infty} \frac{6}{n^6}$ is Convergent by using the Limit Comparison Test.

Note that this work will be a sample proof of the fact that *Constant multiple of a Convergent series* **is** *Convergent.*

12. Show that $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ Diverges using **two** Different methods.

REGULAR OFFICE HOURS

Sunday: 6–7:30 pm TA Nico, SMUDD 207

Monday: 1:00–3:00 pm

6–7:30 pm TA Daksha, SMUDD 207

7:30–9:00 pm TA Karime, SMUDD 207

Tuesday: 12:00–4:00 pm

6–7:30 pm TA Ian, SMUDD 207

7:30–9:00 pm TA Nico, SMUDD 207

Wednesday: 1:00-3:00 pm

9–10:30 pm TA Daksha, SMUDD 207

Thursday: none for Professor

6–7:30 pm TA Ian, SMUDD 207

7:30–9:00 pm TA Karime, SMUDD 207

Friday: 12:00–2:00 pm

Train your Convergence Tests Daily