Math 121 Midterm Exam #2 March 31, 2020

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.
- Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, or $\arctan(\sqrt{3})$ should be simplified.
- Please *show* all of your work and *justify* all of your answers.
- You may work for no more than 3 consecutive hours. You MUST be in front of the ZOOM camera.
- When done, immediately upload to the EXAM 2 entry in Gradescope. **TAG** problems.

1. [40 Points] Compute the following integrals. Justify your work.

(a) $\displaystyle\int_0^5 \frac{6}{x^2-4x-5}\,dx$ You can use this **free** given P.F.D. fact: $\displaystyle\frac{6}{x^2-4x-5} = \frac{1}{x-5} - \frac{1}{x+1}$

(b) $\displaystyle\int_6^\infty \frac{6}{x^2-4x-5}\,dx$ Use the same free P.F.D. fact above.

(c) $\displaystyle\int_{-\infty}^5 \frac{6}{x^2-4x+7}\,dx$

(d) $\displaystyle\int_0^e \ln x\,dx$

(e) $\displaystyle\int \frac{x^3+4x+1}{x^2+1}\,dx$

2. [8 Points] Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{ \left(\frac{n+1}{n}\right)^n \right\}_{n=1}^{\infty}$$

3. [8 Points] Find the **sum** of the following series (which does converge).

$$\sum_{n=1}^{\infty} (-1)^n \frac{4^{n+1}}{3^{2n-1}}$$

4. [20 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{6}{n^6} + \frac{1}{n^6+6}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{6}{\arctan n}$

(c) Use the Absolute Convergence Test to Prove that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^6}$ is convergent.

5. [24 Points] Determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{6n-2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, n^n \cdot n!}{(2n)!}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^3+6}{n^6+3}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 It can be shown that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \ln 2$

Compute the following sum. Justify. $\displaystyle 1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} + \ldots$